Engineering Physics (BT-201)
B.Tech. I Year

Unit I - QUANTUM PHYSICS

OUTLINE

- ➢ WAVE PARTICLE DUALITY
- ➢ GROUP AND PARTICLE VELOCITIES
- ➢ UNCERTAINTY PRINCIPLE
- ➢ ELEMENTARY PROOF AND APPLICATIONS (Uncertainty Principle)
- ➢ COMPTON SCATTERING
- ➢ ENERGY AND MOMENTUM OPERATORS
- ➢ TIME DEPENDENT AND TIME INDEPENDENT SCHRÖDINGER WAVE EQUATION
- ➢ ONE DIMENSIONAL SQUARE POTENTIAL WELL

QUANTUM MECHANICS

We know that classical mechanics can successfully explain the motion of astronomical bodies (such as stars, planets satellites etc.) means Newton‟s law of motion, as well as macroscopic bodies (such as motion under). Except this motion of charged particles in e.m. fields, elastic vibrations in solids, propagation of sound waves in glass etc. can also be explained successfully by classical mechanics, but some phenomenon like black body radiation, photo-electric effect, Compton effect, specific heat of solids at low temperature, stability of atoms, emission and absorption of light etc. could not be explained, which is explained by quantum mechanics.

Wave particle duality or dual nature of light

Light obeys the phenomena of interference, diffraction, polarization, photoelectric effect, Compton Effect etc. The phenomena of interference, diffraction and polarization can be explained by assuming that light is a form of wave. By the wave theory of light, it has been proved that light possesses a wave nature. However, some other phenomena like photoelectric effect, Compton Effect and discrete emission and radiation can be explained only with the help of the quantum theory of light. According to the quantum theory, light radiation travels in the form of energy bundles called quanta of energy hv, where v is the frequency of radiation. Hence according to the quantum theory, light possesses a corpuscular (particle) nature. Therefore, sometimes light obeys the wave theory and sometimes the corpuscular theory, Hence, light has dual nature.

DE-BROGLIE HYPOTHESIS OR DE-BROGLIE MATTER WAVES

Louis De-Broglie suggested that the dual nature is not only of light, but each moving material particle has the dual nature. He assumed a wave should be with each moving particle, (all micro particles) which is called the matter waves. Although these waves can travel through vaccum like e.m. waves, but these are different from

e.m. waves, because these waves associated with all types of charged and neutral particles.

PROOF OF DE-BROGLIE WAVES

According to Plank‟s quantum theory light is in form of small bundles of energy ($h\upsilon$) called quanta or photons.

If we consider a photon (quantum) to be a wave of frequency υ then its energy

$E=h\upsilon$ ....*(i)* Or $E=hc/\lambda$ [i.e. $c=\upsilon\lambda$]

Where c = velocity of light (or photon) in vaccum

λ = wavelength of photon or radiation

h = Plank‟s constant = 6.625×10^{-34} J-sec

Now if we consider photon as a particle of mass m, then from Einstein‟s mass-energy equivalence (or theory of relativity) energy of photon

$E= mc^2$ ....*(ii)* from eq.(i) & (ii) $mc^2 = h\upsilon = hc/\lambda$

or $mc = h/\lambda$

or $\lambda = h/mc$ [i.e. $p = mc$]

or $\lambda = h/p$............*(a)* where λ is a De-Broglie wavelength.

Thus this is an evidence of De-Broglie nature of radiation (photon) the wavelength (λ) to particle like nature of radiation (photon) the momentum (p). It means this equation shows dual nature of light.

We know that the kinetic energy of the particle

$$E = \frac{1}{2}(mv^2) = \frac{1}{2}(m^2v^2/m) = P^2/2m$$

$$\text{or } P^2 = 2mE$$

$$\text{or } P = \sqrt{(2mE)}$$

So, De-Broglie wavelength $\lambda = h/\sqrt{2mE}$..........*(b)*

According to kinetic theory of gases the average kinetic energy of the material particle

$$\frac{1}{2}(mv^2) = 3/2kT$$

$$\text{Or } m^2v^2 = 3mkT$$

$$\text{Or } P^2 = 3mkT$$

$$\text{Or } P = \sqrt{3mkT}$$

So De-Broglie wavelength $\lambda = h/\sqrt{3mkT}$............*(c)*

where k = Boltzmann‟s constant = $1.38\text{X}10^{-23}$ J/K

T = Absolute temperature

If an particle accelerated through a potential difference of V volts, then

Work done by electric field = increase (gain) in kinetic energy

$$\text{or } qV = \frac{1}{2} mv^2$$

$$\text{or } mqV = \frac{1}{2} m^2v^2$$

$$\text{or } 2mqV = m^2v^2 = P^2$$

$$\text{or } \sqrt{2mqV} = P$$

So, De-Broglie wavelength $\lambda = h/\sqrt{2mqV}$

In case of electron wavelength of wave associated with the moving electron

$$\lambda = h/\sqrt{2meV}$$

$$\text{or } \lambda = \sqrt{(150/V)}\ \text{Å} = 12.27/\sqrt{V}\ \text{Å}.$$

PROPERTIES OF DE-BROGLIE WAVES

1. We know that the wavelength of matter waves (de-Broglie waves) associated by a moving particle is $\lambda = h/mv$

   It means $\lambda \alpha$ (1/m) and $\lambda \alpha$ (1/v).

   For a particle at rest means $v = 0$ so $\lambda = \infty$ and if $v = \infty$ then $\lambda = 0$. Here $\lambda = 0$ means that the matter waves are generated only when the particles are in motion.

2. Matter waves are independent of charge because it generated by any moving particle.

3. Energy of particle is given by $E=h\nu$ or $\nu=E/h$, where ν is the frequency of wave.

   We know that $E= mc^2$, where c is the velocity of light.

   So we can write $\nu= mc^2/h$……………….(1)

   We know that wavelength of a wave associated with the particle of mass m, moving with velocity v is, $\lambda = h / mv$ …………………..…(2)

   If de-Broglie's wave velocity (phase velocity) is v_p, then

   $v_p= \nu\lambda$

   or $v_p = mc^2/h * h / mv$

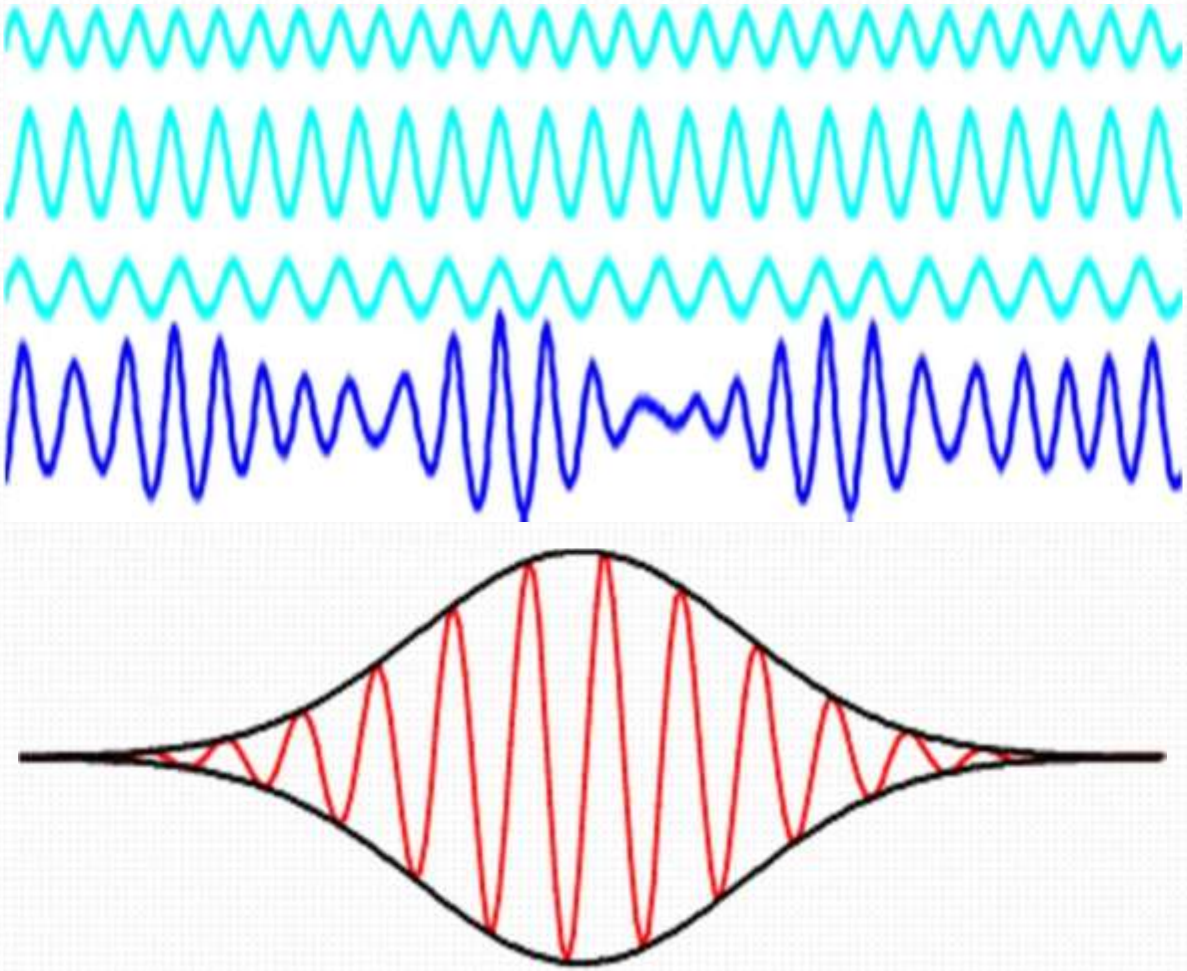
   or $v_p = c^2/v$

- From Einstein‟s theory of velocity, the speed of light is maximum speed that can be attained by a particle in nature.
- Thus from equation $v_p = c^2/v$, the velocity of de-Broglie‟s wave associated with the particle would travel faster than the particle itself,
- Hence it is evident that a particle will not equivalent
to a single wave, but equivalent to a group of wave, called wave packet or wave group.

Wave- Packet or Relation B/W group velocity and phase velocity

- A wave packet consist of a group of several waves of slightly different velocities & wavelength and formed by the superposition of waves situated on and around the center wavelength given by the de-Broglie formula.
- The amplitude and phase of the component waves are such that they interfere constructively in limited region where the particle is found and outside this region they interfere destructively, so amplitude falls to zero rapidly.
- Thus when several waves of slightly different wavelength travel along a straight line in one direction. The resultant waves obtained due to their superposition in form of group of waves which is called the wave packet.

Wave-Packet



The velocity of component or constituents waves of a wave packet is called phase velocity and the velocity of the wave packet is called the group velocity.

Consider the wave group arise from the combination of two waves that have the same amplitude A, but differ by an amount Oω in angular frequency and an amount OK in wave number. These two waves can be represented as,

$y_1 = A\cos(\omega t - kx)$

$y_2 = A\cos[(\omega + O\omega)t - (k + Ok)x]$

Where $\omega = 2\pi\upsilon$ is angular frequency & $k = 2\pi/\lambda$ is wave number (propagation constant)

The motion of wave packet or the displacement equation of wave packet obtained due to their superposition will be at any point „x‟ and time „t‟:

CONT..

$Y = y_1 + y_2$

or $Y = A\,[\cos(\omega t - kx) + \cos\{(\omega + O\omega)t - (k + Ok)x\}]$

or $Y = 2A\cos\dfrac{(\omega t - kx) + \{(\omega + O\omega)t - (k + Ok)x\}}{2} . \dfrac{\cos[(\omega t - kx) - \{(\omega + O\omega)t - \{(k + Ok)x\}]}{2}$

[Because $\cos C + \cos D = 2\cos\{C+D\}/2 . \cos\{C-D\}/2$]

Or, $Y = \dfrac{2A\cos[(\omega t - Kx + \omega t + O\omega t - Kx - OKx)]}{2} \times \dfrac{\cos[(\omega t + O\omega t - Kx - OKx - \omega t + Kx)]}{2}$

Or, $Y = \dfrac{2A\cos[2(\omega t - Kx) + (O\omega t - OKx)]}{2} \times \dfrac{\cos(O\omega t - OKx)}{2}$

Or, $Y = \dfrac{2A\cos\,[(2\omega + O\omega)\,t - (2K + OK)x]}{2} \times \dfrac{\cos(O\omega t - OKx)}{2}$

Because $O\omega << \omega$ and $OK << K$, so we can write $2\omega + O\omega = 2\omega$ and $2K + OK = 2K$

Or, $Y= 2A \cos[(2\omega t-2Kx)/2 \times \cos(O\omega t-OKx)/2]$ CONT..

$$Y= 2A \cos(\omega t-Kx) \times \cos(O\omega t-OKx)/2$$

Or, $Y= 2A \cos(O\omega t-OKx)/2 \times \cos(\omega t-Kx)$

This equation represents a wave of angular frequency „ω‟ & propagation constant „K‟. The amplitude of this wave is $2A \cos(O\omega t-OKx)/2$
The velocity of group wave (vg) is the velocity with which the maximum amplitude moves and maximum amplitude will be „2A‟ i.e.,
$2A \cos(O\omega t-OKx)/2 = 2A$

It means $\cos(O\omega t-OKx)/2 = 0$ means $O\omega t-OKx = 0$

Or $O\omega t=OKx$, or $(O\omega/OK)t = x$

Because we know that x/t= v (distance/ time = velocity). So in case of wave packet group

CONT..

velocity $v_g = \frac{dx}{dt} = \frac{d}{dt}\frac{O\omega t}{OK} = O\omega / OK$

Or, we can write $v_g = \frac{d\omega}{dK}$ ----------------------------(1)

Because when $= \lim_{OK \to 0} \left(\frac{O\omega/2}{OK/2}\right)$

Because the displacement equation of a wave $y= A \cos (\omega t\text{-}Kx)$

So, phase or wave velocity $v_p = dx/dt$

So, A will be maximum when $\cos (\omega t\text{-}Kx)=1$ Or, $\omega t\text{-}Kx=0$ or $x= \omega t/K$

So, phase velocity $v_p = \frac{dx}{dt} = \frac{d}{dt}\frac{\omega t}{K}$ or $v_p = \omega/K$ -------------------------(2)

CONT..

So, from equation (1) $v_g = d\omega/dK$

Means $v_g = \frac{d}{dK}(Kv_p)$ $\therefore v_p = \omega/K$

or $v_g = v_p \times 1 + K . dv_p/dK$

or $v_g = v_p + (2\pi/\lambda) . dv_p/d(2\pi/\lambda)$

or $v_g = v_p - (2\pi/\lambda) . \frac{dv_p}{(2\pi/\lambda^2)d\lambda}$ $\therefore d(1/x) = -(1/x)dx$

or $v_g = v_p - \frac{\lambda . dv_p}{d\lambda}$

Thus group velocity (v_g) depends on the phase velocity (v_p) and variation of phase velocity with the wavelength ($dv_p/ d\lambda$).

DIFFERENT CASES

In the non-dispersive medium: If $dv_p/ d\lambda = 0$ i.e., the medium is such that in it the phase velocity doesn't depend on the wavelength then,

$$v_p = v_g$$

i.e., the group velocity and phase velocity are equal. Such a medium is which the $v_p = v_g$ is called non-dispersive medium. For example; $v_p = v_g$ for electromagnetic wave in vacuum and elastic wave in a homogenous medium.

In the dispersive medium: In a dispersion medium $dv_p/ d\lambda$ is positive and $v_g < v_p$. For example; v_g of electromagnetic wave in a dielectric substance is less than v_p .

But if the value of $dv_p/ d\lambda$ is negative in a dispersive medium , then $v_p > v_g$. For example; in electric conductors the group velocity v_g is more than the phase velocity v_p.

Relationship between group velocity & phase velocity for the De-Broglie wave associated with a particle of rest mass (m_o) moving with a velocity v).

➢ (Relativistic Case)

Because we know that angular frequency ,

$\omega = 2\pi\upsilon = 2\pi.(mc^2/h)$ [$.^{\cdot}.$ E= m = hυ or υ = mc^2/h]

Or $\omega = \frac{2nn_0c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$------------(1) $\left[.^{\cdot}. \quad n = \frac{n_0}{\sqrt{1-\frac{v^2}{c^2}}}\right]$

And propagation constant $K = 2\pi/\lambda = 2\pi.(m\upsilon/h)$ [$.^{\cdot}.$ λ=h/p=h/mv]

$K = \frac{2nn_0v}{h\sqrt{1-\frac{v^2}{c^2}}}$ ------------(2)

So phase velocity $v_p = \omega/K = c^2/v$, and

Graph velocity $v_g = \dfrac{d\omega}{dK} = \dfrac{d\omega/dv}{dK/dv}$

Now $\dfrac{dm}{dv} = \dfrac{d}{dv}\left[\dfrac{2nn_0c^2}{h\sqrt{1-\frac{v^2}{c^2}}}\right]$ or $\dfrac{d}{dv}\left[\dfrac{2nn_0c^2}{h}\left(1-\dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]$

$\dfrac{dm}{dv} = \dfrac{2nn_0c^2}{h}\left[\dfrac{d}{dv}\left(1-\dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]$ After Differentiation

$\dfrac{dm}{dv} = \dfrac{2nn_0c^2}{h}\left[\dfrac{v}{c^2}\left(1-\dfrac{v^2}{c^2}\right)^{-\frac{3}{2}}\right]$ Similary for $\dfrac{dK}{dv} = \dfrac{2nn_0}{h}\left[\left(1-\dfrac{v^2}{c^2}\right)^{-\frac{3}{2}}\right]$

So we can write Graph velocity $v_g = \frac{dm}{dK} = \frac{dm}{dv} / \frac{dK}{dv} = v$

$$v_p = \frac{c^2}{v_g}$$

So, the De – Broglie wave group associated with a moving body travels with the same velocity as the moving particle. It is evident that a material particle in motion is equivalent to a group of waves or a wave packet.

Relation between Vp and Vg for a non- relative free particle (Non relativistic case)

Suppose v_g & v_p represents the group and phase velocity respectively for a non-relativistic free particle of mass 'm'.

Let 'λ' is the De- Broglie wavelength and 'υ' is the frequency of the wave then, the phase velocity, $v_p = \upsilon\lambda$--------------------(1)

According to De-Broglie hypothesis, $\lambda = h/mv_g^2$,-----------------------(2)

Total energy, E= kinetic energy = $mv_g^2/2$

[Because potential energy of freely moving particle is constant]

Also E= $h\upsilon$. Or, $\upsilon = E/h = mv_g^2/2h$ -----------------------(3)

So from equation (1), (2) and(3),

Because phase velocity $v_p = \upsilon.\lambda = mv_g^2/2h \times h/mv_g^2$

$$v_p = v_g /2$$

Hence for a non- relativistic free particle the phase velocity is half of the group velocity.

Heisenberg‟s Uncertainty Principle

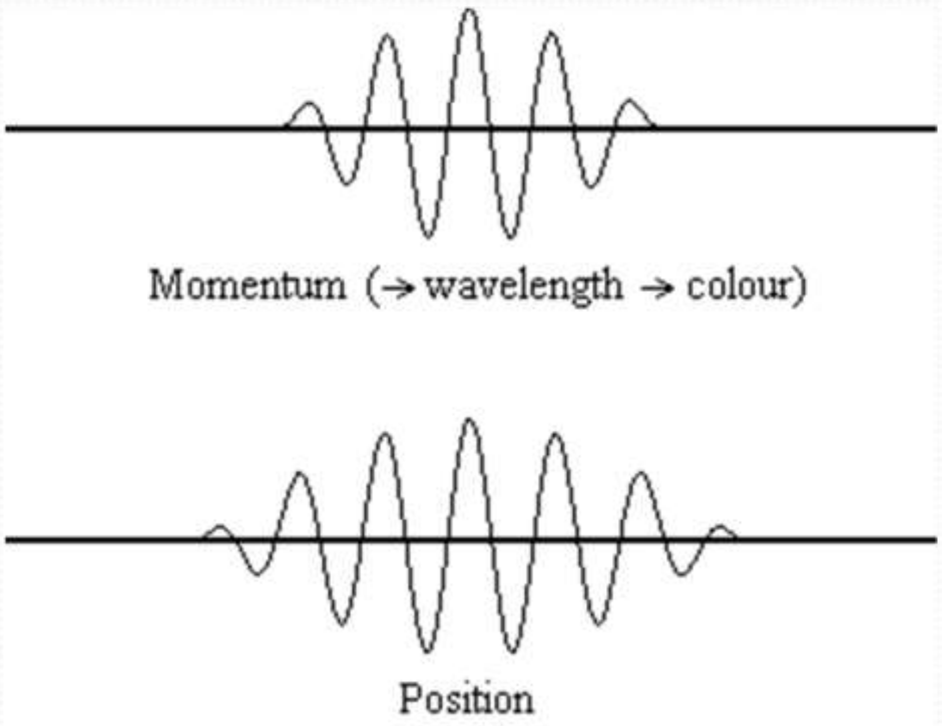
Werner Heisenberg

"It is impossible to determine simultaneously the position and momentum of the particle with any desired accuracy."

This definition is known as Heisenberg‟s uncertainty principle.

- This limitation is critical when dealing with small particles such as electrons.
- But it does not matter for ordinary-sized objects such as cars or airplanes.
- To locate an electron, you might strike it with a photon.
- The electron has such a small mass that striking it with a photon affects its motion in a way that cannot be predicted accurately.
- The very act of measuring the position of the electron changes its momentum, making its momentum uncertain.

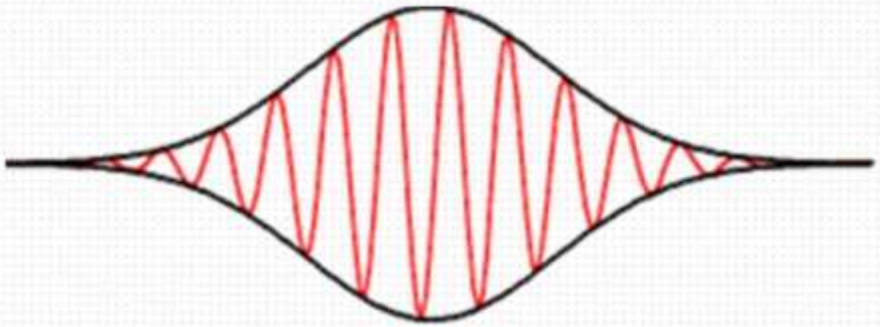
If we want accuracy in position, we must use short wavelength photons because the best resolution we can get is about the wavelength of the radiation used. Short wavelength radiation implies high frequency, high energy photons. When these collide with the electrons, they transfer more momentum to the target. If we use longer wavelength, i.e less energetic photons, we compromise resolution and position.



Proof of Heisenberg"s principle

1. Position and Momentum uncertainty:-

Heisenberg"s principle can be proved by assuming that a particle in motion is equal to wave group and $v_g = v$. We know that a moving particle is equal to a wave group rather than a particle. It means there is limit for the measurement of particle properties.



Let a particle surrounded by a wave group. Let this wave group arises from the combination of two waves that have same amplitude A but differ by an amount $\Delta\omega$ in angular frequency and an amount Δk in propagation constant.

These two waves can be represented as :-

$y_1 = A\cos(\omega t - kx)$

and $y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$

The displacement of wave group at any time & at any point x will be.

$Y = y_1 + y_2$

or $Y = A[\cos(\omega t - kx) + \cos\{(\omega + O\omega)t - (k + Ok)x\}]$

or $Y = 2A\cos\dfrac{[(\omega t - kx) + \{(\omega + O\omega)t - (k + Ok)x\}]}{2} . \cos\dfrac{[(\omega t - kx) - \{(\omega + O\omega)t - \{(k + Ok)x\}]}{2}$

[Because $\cos C + \cos D = 2\cos\{C+d\}/2 . \text{Cos}\{C-D\}/2$]

Or, $Y = 2A\cos\dfrac{[(2\omega + O\omega)t - (2K + OK)x]}{2} \times \cos(O\omega t - OKx)/2$

Because $O\omega << \omega$ and $OK << K$, so we can write $2\omega + O\omega = 2\omega$ and $2K + OK = 2K$

Or, $Y = 2A\cos(O\omega t - OKx)/2 \times \cos(\omega t - Kx)$

This equation represents a wave (wave packet) of angular frequency „ω" & propagation constant „K". The amplitude of this wave is
$2A \cos(O\omega t - OKx)/2$

Because the particle is moving with a velocity equal to the velocity of wave packet so the position of the particle can be anywhere in the wave packet so we can say the uncertainty in the position of the particle equals to the length of wave packet. Means distance between two consecutive modes. Node is formed when amplitude =0, means

$$2A \cos(O\omega t - OKx)/2 = 0$$

Or

$$\cos(O\omega t - OKx)/2 = 0$$

It is possible when $(O\omega t - OKx)/2 = \pi/2, 3\pi/2, 5\pi/2, \ldots$

At a particular time „t‟ at positions x_1 and x_2,

$(O\omega t - OK\, x_1)/2 = \pi/2$ -------------------- (1)

$(O\omega t - OK\, x_2)/2 = 3\pi/2$ ------------------- (2)

From equations (1) and (2), we can write

$OK/2.(x_2 - x_1) = \pi$

If the error in the measurement of the position of the particle is $(x_2 - x_1) = Ox$, then

$OK.\ Ox = 2\pi$ ------------------- (3)

If Op is error in the measurement of the momentum of the particle and OK in propagation constant then,

$OK = 2\pi/O\lambda = (2\pi.Op)/h$ So from Equation (3) we can write,

$(2\pi.Op)/h.\ Ox = 2\pi$ Or $Op.\ Ox = h$ Or $Op.\ Ox \geq \hbar/2\pi$

It means that if we make simultaneous measurement of two quantities namely position and momentum of the particle the product of the fundamental errors is approximately equal to Plank‟s constant.

Application of Uncertainty principle

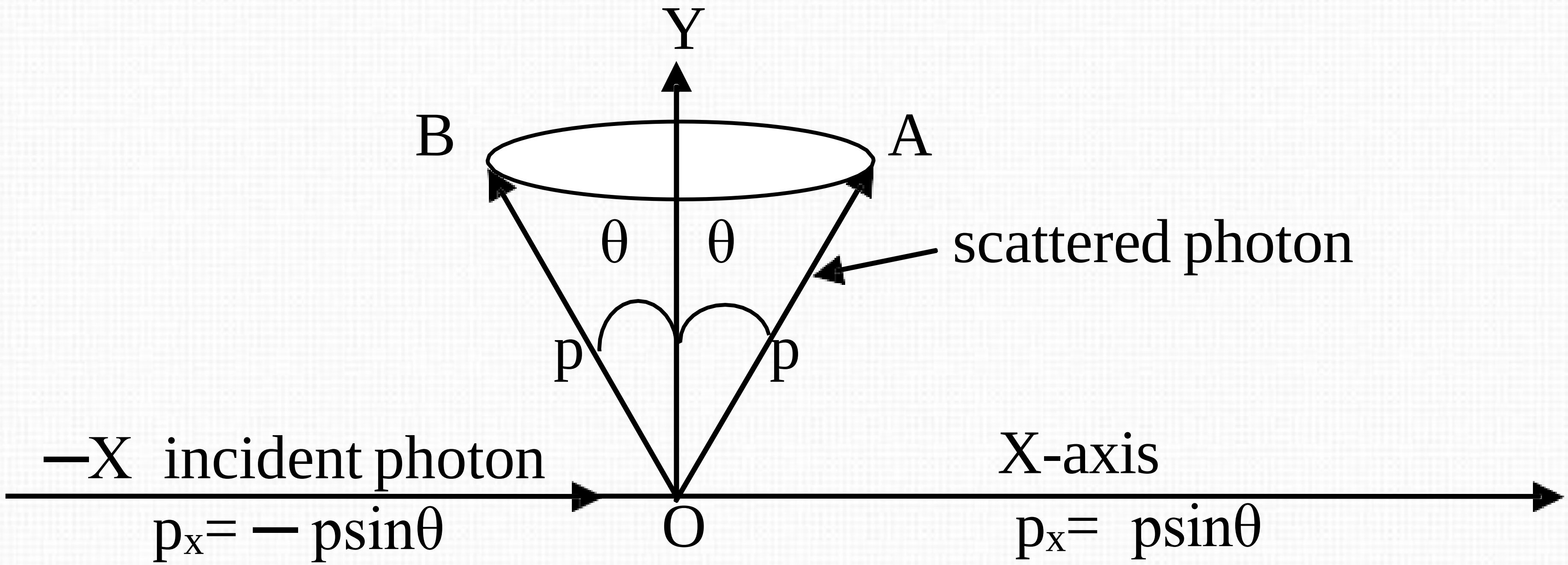
Determination of position of α-particle by γ-ray microscope:

For the determination of position of an particle (electron), it should be targeted (illuminated) by light (photons).From physical optics it is known that the exactness of position increase with decrease in wavelength of light. So uncertainty in position „Ox‟so we take γ-rays microscope to be small, wavelength „λ‟should be small. We know that from the theory of resolving power, the minimum distance between two objects x_1 and x_2 to be seen clearly or the uncertainty in the position is

$(x_2 - x_1) = Ox = \lambda/2\sin\theta$ -------------------------(1)

Where λ is wavelength of γ-rays and θ is semi-vertical angle of the cone of the objective with the object.

Suppose γ-rays incident on a stationary electron at O. The scattered γ-rays to be seen by the microscope which reaches through the objective of the microscope AB. The γ-rays consist of photons of energy $h\upsilon$ and momentum $h\upsilon/c$. Due to collision of photons with electron there will be a transfer of momentum from photon to electron. Therefore change in momentum of photon which will reach the objective of microscope.


Y
B
A
θ
θ
scattered photon
p
p
−X incident photon
X-axis
O
px= − psinθ
px= psinθ

So the momentum of photon in the direction of motion of electron will be p sinθ and in the direction opposite of motion of electron will be psin(-θ).

Total change in momentum Op = psinθ – psin(-θ)

Op = psinθ + psinθ

Op = 2psinθ ---------------- (2)

From equation (1) (2), we get

Op.Ox = λ/2sinθ. 2psinθ

Or Op.Ox = h/p2sinθ.

2psinθ Or Op.Ox = h

Or Op.Ox ≥ ħ/2π

Thus in the determination of position of a particle by γ-ray microscope the product of uncertainty in position and uncertainty in momentum is of the order of h. It is in accordance with Heisenberg‟s Uncertainty principle.

2. Diffraction of an electron beam by a single slit: Consider an electron beam travelling in X-direction which is incident on a narrow slit AB. Since the electron beam has a wave behavior, so we get diffraction pattern on the screen. Because it is quite uncertain to say that from which place of the slit the electron passes. If the width of the slit is Oy, the maximum uncertainty in the position of electron screen = Oy (in Y-direction)

Obviously narrow the slit less the uncertainty in position. But according to the theory of diffraction at a single slit, for half angular width of principal maxima

$Oy.\sin\theta = \lambda$ for the wave of wavelength λ.

Or $\sin\theta = \lambda/Oy$

So maximum uncertainty in the position of electron is,

$Oy = \lambda/\sin\theta$ ....(1)

If the wavelength of wave associated with electron is λ, then from de-Broglie $\lambda = h/p$.

The momentum of electron in parallel to the slit (i.e. in Y- direction) can have any value between $p\sin\theta$ and $p\sin(-\theta)$ because the diffracted electron can be found anywhere within the principle maxima (angular spread from $-\theta$ to $+\theta$).

Therefore uncertainty in momentum in direction parallel to the slit

$\Delta p_y = p\sin\theta - p\sin(-\theta) = p\sin\theta + p\sin\theta = 2p\sin\theta$

$\Delta p_y = 2(h/p).\sin\theta$ ....(2)

From (1) and (2), we get

$\Delta p_y . Oy = \lambda/\sin\theta . 2(h/p).\sin\theta = 2h \geq \hbar$

3. Non-existence of electron in nucleus: From Rutherford"s experiment we know that size of nucleus is equal to 10^{-14}meter. If electron exists in the nucleus then uncertainty in position of electron is Ox is the same as the size of nucleus. It means

$$Ox = 10^{-14} \text{meter.}$$

So minimum uncertainty in momentum $Op_{min} = h/Ox = 6.625\times10^{-34}$j.sec.
10^{-14}meter

Or $Op_{min} = 6.625\times10^{-34}$kg.m²/sec².sec $=6.625\times10^{-20}$kg.m/sec 10^{-14}meter

For electron of minimum momentum inside the nucleus minimum energy $OE_{min}= (p^2c^2 + m_0c^4)^{1/2}$

$OE_{min}= [(3.31\times10^{-20})^2(3\times10^8)^2+(9.1\times10^{-31})^2 (3\times10^8)^4]^{1/2}$

$OE_{min} = [(10.95\times10^{-40})(9\times10^{16})+(82.81\times10^{-61})(9\times10^{32})]^{1/2}$ $OE_{min} = [(98.55\times10^{-24})+(414.05\times10^{-29})]^{1/2}$

Because second term is much smaller than first in above equation, so we can neglect it.

$OE_{min} = 9.93\times10^{-12}$ joule Or $OE_{min} =$ 9.93×10^{-12} eV 1.6×10^{-19}

Or $OE_{min} = 9.93\times10^{-12}$ MeV
$1.6\times10^{-19}\times10^6$

Or $OE_{min} = 52.26$ MeV

Because maximum kinetic energy of a β- particle emitted from radioactive nuclei is of the order of 4 MeV.

Conditions For Acceptable Wave Function

Because $\psi^*\psi = |\psi|^2$ = a real quantity. Thus it is clear that $|\psi|^2$ is a real quantity and is a measure of probability density. Hence probability of finding the particle in a small volume dv = $|\psi|^2 dv = |\psi|^2 dxdydz$Since total probability of finding the particle in any position is unity.

So $\int_{-\infty}^{+\infty} |\psi|^2 dv = 1$ or $\int_{-\infty}^{+\infty} \psi^*\psi dv = 1$

This condition is known as normalization condition. The function satisfied the condition is called normalized wave function.

So ψ must be normalized, single valued because at any instant t there can be only one probability for the particle to be at a point; ψ must be finite and continuous.

Expectation Value: -

The expectation value of a quantity $< f(r) > = \dfrac{\iiint \psi^* f(r) \psi dv}{\iiint \psi^* \psi dv}$

-

$$< f(r) > = \frac{\iiint f(r)|\psi|^2 dv}{\iiint |\psi|^2 dv}$$

If the wave function is normalized then $\iiint f(r)|\psi|^2 dv = 1$, so

$$< f(r) > = \iiint f(r)|\psi|^2 dv$$

Schrodinger"s Wave Equation

It is a differential equation of the de-Broglie waves associated with the particle and describes the motion of particle.

If a wave function associated with a particle which is moving with velocity v in +ve direction, then displacement of wave is given by

$\psi = Ae^{-i\omega(t - x/v)}$ ....

$= Ae^{-i(\omega t - x.2\pi\upsilon/\upsilon\lambda)}$ (1) (Because v = υλ and ω = 2π/λ)

$= Ae^{-i(\omega t - x.2\pi/\lambda)}$

$= Ae^{-i(\omega t - k.x)}$ (Because 2π/λ)

$= Ae^{-i\,2\pi(\upsilon t - x/\lambda)}$

$= Ae^{-i\,2\pi(Et/h - xP/h)}$ (Because E = hυ and λ = h/p)

$= Ae^{-i\,2\pi/h\,(Et - P.x)}$

$= Ae^{-i/\hbar\,(Et - P.x)}$ ....(2)

This is a wave equation for a freely moving particle. Now differentiating equation (2) with respect to get

$\partial\psi/\partial t = -(iE/\hbar).Ae^{-i/\hbar\,(Et - P.x)}$

$\partial\psi/\partial t = -(iE/\hbar).\Psi$ (Because $\psi = Ae^{-i/\hbar\,(Et - P.x)}$)

$\partial\psi/\partial t = -(i/\hbar).E\psi$

or $E\psi = (i\hbar)\,\partial\psi/\partial t$ ....(3)

$E\psi = -(\hbar/i)\,\partial\psi/\partial t$

So energy operator $E = (i\hbar).\partial/\partial t$ ....(4)
Now partially differentiating equation (2) with respect to x, we get

$$\partial\psi/\partial x = (iP/\hbar).\ Ae^{-i/\hbar\,(Et - P.x)}$$

$$\partial\psi/\partial x = (iP/\hbar).\Psi \qquad (\text{Because } \psi = Ae^{-i/\hbar\,(Et - P.x)}$$

$$) \text{ or } P\psi = (\hbar/i).\partial\psi/\partial x$$

$$\text{or } P\psi = (-i\hbar).\partial\psi/\partial x \quad \text{............................(5)}$$

So momentum operator $P = (-i\hbar).\partial/\partial x$
............................(6) We know that total energy of the particle

$$E = K.E. + P.E.$$

` $E = mv^2/2 +$
V or $E = p^2/2m$
$+ V$

Total energy in terms of wave function or operating wave function on above equation, $E\psi = (p^2/2m)\,\psi + V\psi$ ....(7)
Putting the value of E & P from equation (4) & (6) we get, $[(i\hbar).\partial/\partial t\,]\psi = [(-i\hbar).\partial/\partial x\,]^2\psi/2m + V\psi$

$$(i\hbar).\partial\psi/\partial t = [(\hbar^2/-1).\partial^2/\partial x^2\,]\psi/2m +$$

$$V\psi \quad (i\hbar).\partial\psi/\partial t = (\hbar^2/-2m).\partial^2\psi/\partial x^2 + V\psi$$

$$(i\hbar).\partial\psi/\partial t = -(\hbar^2/2m).\partial^2\psi/\partial x^2 + V\psi$$

This is Schrodinger time dependent equation for one dimension. For three dimensional case, $(i\hbar).\partial\psi/\partial t = -\,(\hbar^2/2m).[\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2]\,\psi + V\psi$
or $(i\hbar).\partial\psi/\partial t = -\,(\hbar^2/2m).\quad \psi + V\psi$ (Where $\quad = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$)

For free moving particle $V = 0$ so we can write, $(i\hbar).\partial\psi/\partial t = -(\hbar^2/2m).$

ψ

Now putting the value of P from equation (4) to equation (7) we get,

$$E\psi = -(\hbar^2/2m).\partial^2\psi/\partial x^2 + V\psi$$

$$\text{or } (\hbar^2/2m).\partial^2\psi/\partial x^2 + (E - V)\psi = 0$$

$$\text{or } \partial^2\psi/\partial x^2 + (2m/\hbar^2).(E - V)\psi = 0 \quad \text{.............................(8)}$$

This is Schrodinger time independent equation for one dimension. For three dimensional case, $[\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2]\psi + (2m/\hbar^2).(E - V)\psi = 0$

$$\psi + (2m/\hbar^2).(E - V)\psi = 0$$

For free moving particle $V = 0$ so we can write,

$$\psi + (2m/\hbar^2).E\psi = 0$$

Q. Show that the function $\psi = Axe^{(-x^2/2)}$ is the eigen function of the operator A


Thank You!

UNITII **WAVEOPTICS**

Introduction:

Optics isa branchofphysicswhichdealswiththe "Theoryoflight and itspropagation in a given medium". The branch of optics is divided into two parts

1. RayorGeometricalopticsand

2. PhysicalorWaveoptics

Ray optics: Ray optics deals with image formation byoptical systems. It was supported by Newton'scorpusculartheory. Itdealswiththeparticlespropagating ina mediumcalled "light- photons with energy $E_n = nh\upsilon$ in a medium"

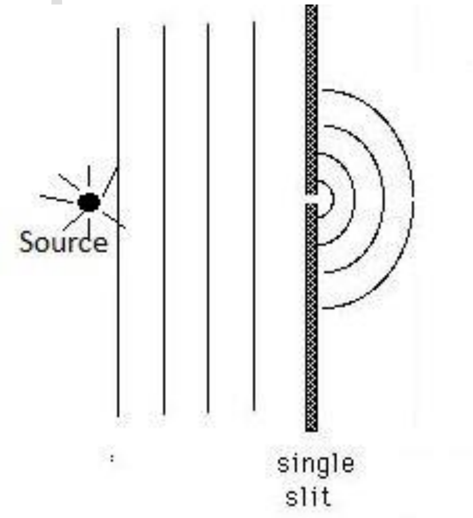
Physical optics: Physical optics deals with nature of light. Huygens proposed the wave theoryoflight. Accordingtothis, a luminousbodyisasourceofdisturbance inahypothetical medium called "ether"

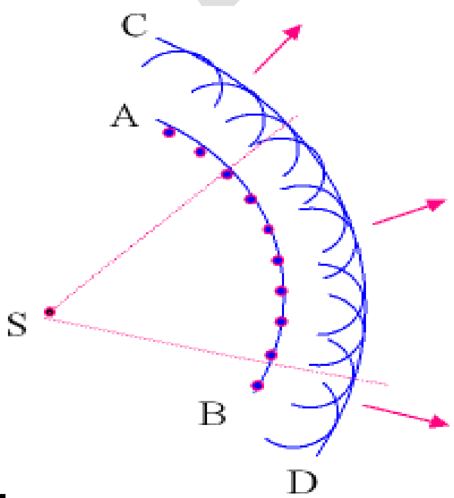
HUYGEN'SPRINCIPLE

Huygensprincipleprovidesageometricalmethodoffindingtheshapeandpositionofthe wave front at a certain instant from its shape and position of some earlier instant.
Huygen'sprincipleisstatedinthefollowingtwo parts.

i. Each point on the wave front acts as a centre of new disturbance and emits its own set of spherical waves called secondary wavelets. These secondary wavelets travel in all directions withthe velocityoflight so long as they move in the same medium.

ii. Theenvelopeorthe locusofthese waveletsin the forward direction gives the position ofthe new wavefront at any subsequent times.





Huygens' Principle:
Each wavefront is the envelope of the wavelets. Each point on a wavefront acts as an independent source to generate wavelets for the next wavefront. AB and CD are two wavefronts.

Superposition Principle: According to superposition principle "The resultant or total displacement ofthe medium acted upon bytwo or more waves simultaneously equal to sumor difference of displacements of individual waves".

$$R or Y = y_1 \pm y_2 \pm \text{----------} \pm y_n$$

Interferenceoflight:

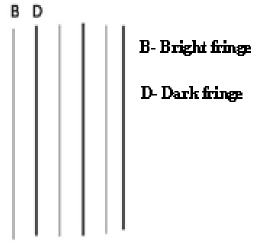


Fig: Interference fringe pattern

Itisdefinedas"ModificationofResultantIntensityof lightobtainedbythesuperpositionoftwoormorelightwaves". Thistheoryofinterferenceoflightwasdevelopedby Thomas youngin his experimental study of light. The resultantintensity consistsofseriesofbright&darkfringesthatappearonthe screenwhichareknownasinterferencepatternorinterference fringeswhichcorrespondto maximum&minimumintensitiesoflight.

Interferencepatternmaybeeitherstraightorcircularorparabolic(Arc)fringes

CoherentSources: Ifthephasedifferencebetweentwolight wavesemittedfromtwo sources is zero or has a constant value then the sources are said to be coherent.

Incoherent-sources: Ifthephasedifference betweentwolightwavescoming fromtwo sourceschangeswithtime,thesourcesarecalledas"In-coherentSources".

Typesofinterference: Interferenceoflight baseduponyoung'sdoubleslitexperimentis divided intotwotypes.

1. Constructiveinterference
2. Destructiveinterference

Constructiveinterference: Whenthetwo light wavesreachapoint inphasetheresultant displacement (R_{Max} or Y_{Max}) is always equal to algebraic sum of individual displacements of the light waves. It is known as constructive interference.

$$R_{Max} or\ Y_{Max} = y_1 + y_2 + \text{\_\_\_\_\_\_} + y_n$$

Destructive interference: When the two light waves reach a point i.e. out of phase. The resultant displacement (R_{Min} or Y_{Min}) isalwaysequaltothedifferenceofdisplacementsofthe light waves. It is known as destructive interference.

$$R_{Min} or Y_{Min} = y_1 - y_2 \text{\_\_\_\_\_\_\_\_} - y_n$$

Conditionsforinterferenceoflight:

1) Thetwolight sourcesemittinglightwavesshouldbecoherent.
2) Theseparationbetweenthetwosourcesshouldbesmall.
3) Thedistance betweenthetwosourcesandthescreenshouldbelarge.

4) Toviewinterferencefringes,theback-ground(good-contrast)shouldbedark.
5) Amplitudesoftwolightwavesarenearlyequal.
6) Thesourceshouldbe mono-chromatic.
7) Thesourcesshouldbe narrowi.e.theymustbesmall.

Interferenceinthinfilmsduetoreflectedlight:

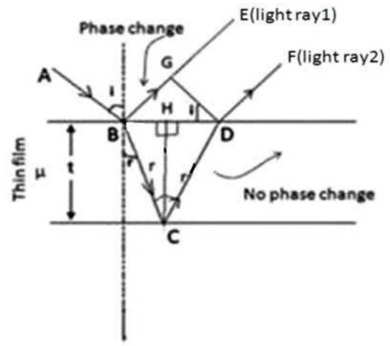


Fig-1: Interference in thin films due to reflection

When light is incident on a (plane parallel) thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again, some portionofthetransmittedlight isreflectedback intothe filmby lower surface and emerges through the upper surface. These reflected light beams superimpose with each other, producing interference and forming interference patterns.

Consider a thin film of thickness‘t’ and refractive index μ. Let a monochromatic light rayAB be incident at an angle of incidence of‘i’ on the upper surface ofthe film. BE and BC arethereflectedandtransmittedlight rays. Lettheangleofrefractionis‘r’. TherayBCwillbe reflected into the film and emerge throughthe film in the form of the light rayDF.

These two light rays superimpose depending upon pathdifference between them producing interference patterns. To knowthe pathdifference, let us drawthe normalDG to BE. From D and G onwards, the light rays travel equal distances. By the time the light ray travels from B to G, the transmitted light rayhas to travel from B to C and C to D.

Thepathdifferencebetweenlightrays(1)and(2)is

$$\Delta = \mu(BC+CD)\ (\text{infilm}) - BG(\text{inair}) \text{ ---------------(1)}$$

FromtriangleBCH, $$BC = \frac{HC}{\cos r} = \frac{t}{\cos r}$$

$$\therefore BC = \frac{HC}{\cos r} = \frac{t}{\cos r}$$

FromtriangleDCH, $$\cos r = \frac{HC}{CD}$$

$$\Rightarrow \cos r = \frac{t}{CD}$$

$$CD = \frac{t}{\cos r}$$

$$BC = CD = \frac{t}{\cos r}$$

$$\Rightarrow (BC+CD)=\frac{2t}{\cos r} \quad \text{--------(2)}$$

TocalculateBG, BD=BH+HD

$$\text{TriangleBHC}, \tan r=\frac{BH}{CH}=\frac{BH}{t}$$

∴BH=ttanr

SimilarlyHD=ttanr

BD=BH+HD=2ttanr (∵BH=HD)

$$\text{FromtriangleBGD}, \sin i=\frac{BG}{BD}$$

$$\Rightarrow BG=BD\sin i$$

$$\therefore BG=2t\tan r\sin i$$

FromSnell'slaw, $\mu=\frac{\sin i}{\sin r}$

$$\sin i=\mu\sin r$$

$$BG=2\mu t\tan r\sin r$$

Substitutingtheabovevaluesinequation-(1)

$$pathdifference=\mu\left(\frac{2t}{\cos r}\right)-2\mu t(\tan r)\sin r$$

$$=\frac{2\mu t}{\cos r}\left(1-\sin^2 r\right)$$

$$=\frac{2\mu t}{\cos r}\left(\cos^2 r\right)$$

$$\boxed{pathdifference=2\mu t\cos r}$$

Intheabovecase,therayBEisreflectedatthesurfaceofadensermediumandhenceit suffersanadditionalpathdifferenceequalto $-\frac{\lambda}{2}$.

TherayBCsuffersreflectionat C,atthesurfaceofararer mediumand henceon emerging out at D, it suffers no additional path change.

HencethenetpathdifferencebetweenthetworeflectedraysDFandBE

$$\text{Total path difference} = 2\mu t\cos r + \frac{\lambda}{2}$$

When the path difference is an integral multiple of λ then the rays (1) and (2) will be in phase and appear as constructive interference.

$$\text{i.e., } 2\mu t\cos r + \frac{\lambda}{2} = n\lambda \qquad \text{(Condition for the bright fringe)}$$

$$2\mu t\cos r = n\lambda - \frac{\lambda}{2}$$

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2} \quad \text{where } n = 0, 1, 2 \ldots\ldots\ldots$$

When the path difference is half integral multiple of λ, the rays (1) and (2) meet in out of phase and undergo destructive interference.

$$\text{i.e., } \quad 2\mu t\cos r + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2} \qquad \text{(Condition for the dark fringe)}$$

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t\cos r = n\lambda \quad \text{where } n = 0, 1, 2 \ldots\ldots\ldots$$

Depending on the conditions the interference pattern consists of bright and dark fringes.

Newton's Rings:

Newton's rings are one of the best examples for the interference in a non uniform thin film. When a plano convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero.

If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are seen. These circular rings were discovered by Newton and are called as Newton's rings.

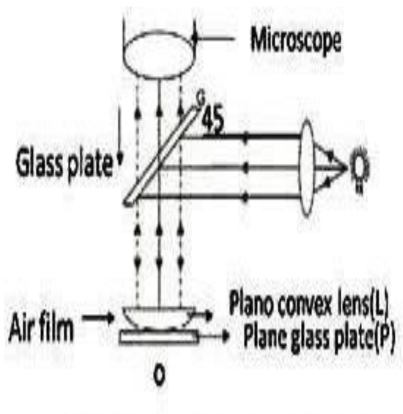


Fig 2 Newton's rings experiment

The plano-convex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate (P). The lens makes the contact with the plate at 'O'. The monochromatic light falls on a glass plate G held at an angle of 45^0 with the vertical. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P. A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface ofthe plate. These reflected rays interfere and give rise to an interference patterninthe form of circular rings. These rings are seen near the upper surface of the air film through the microscope.

Explanation:

Newton'sringsare formed due to interference betweenthe light raysreflected fromthe topandbottomsurfacesofair filmbetweentheplateandthe lens. Apartofthe incident light is reflected ata pointin the form of the ray (1) with any additional phase change. The other part is refracted and again reflected in the form of the ray (2) with additional phase change of π or path change of λ/2.

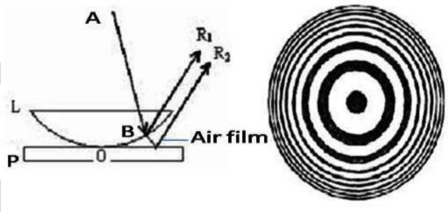


Fig: formation of Newton's rings

Astheringsareobservedinthereflectedlight, the path difference between the rays is

$$pathdiffernce = 2\mu t \cos r + \frac{\lambda}{2}$$

Forair filmμ=1andfor normalincidencer =0,

then, $$pathdifference = 2t + \frac{\lambda}{2}$$

Atthepointofcontactt =0thepathdifference isλ/2, i.e.,thereflected light atthepoint ofcontact suffers phase change ofπ. Then the incident and reflected rays are out ofphase and interface destructively. Hence the central spot is dark.

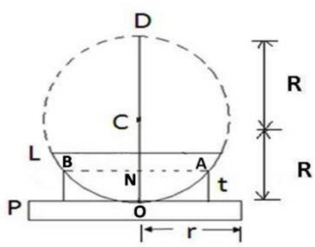
$$\text{Theconditionforthebrightringis} \; 2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{wheren=1, 2,3…..}$$

$$\text{Theconditionfor darkringis} \; 2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{wheren=0,1,2,3……}$$

Theory:



Tofindthediametersofdarkand bright rings,let 'L'bethe lensplaced ona glassplate P.The convexsurface ofthe lens isthe part of spherical surface with centre at 'C'. Let R be the radius of curvatureand rbetheradiusofNewton'sring corresponding tothefilmthickness't'.

Fromthepropertyofacircle,

$$NA \times NB = NO \times ND$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

whereRistheradiusofcurvatureofplanolensand't'isthemaximumthicknessofairfilm

As't'issmall,r^2 willbenegligible, $r^2 = 2Rt$

$$t = \frac{r^2}{2R}$$

Conditionforbrightringis, $2t = (2n-1)\frac{\lambda}{2}$

$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

ReplacingrbyD/2thediameterofnthbrightringwill be

$$\frac{D_n^2}{4R} = \frac{(2n-1)\lambda}{2}$$

$$\therefore D \alpha \sqrt{oddnaturalnumber} \quad \text{(forbrightring)}$$

$$D \alpha \sqrt{naturalnumber} \quad \text{(fordarkring)}$$

Thus, the diameters ofdark rings are proportional to the square root ofnatural numbers and diameters of bright rings are proportional to odd natural numbers.

Determinationofwavelengthofalightsource:

Let R be the radius of curvature of a Plano convex lens, λ be the wavelength of light used. Let $D_m$and $D_n$are the diameters of mth and nth dark rings are respectively.

$$D_m^2 = 4m\lambda R$$

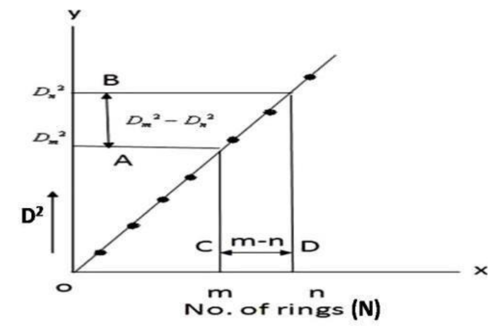


Fig: Plot of D^2 w.r.t. no. of rings

$$D_n^2 = 4n\lambda R$$

$$D_m^2 - D_n^2 = 4(m-n)\lambda R$$

$$\lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}$$

$$R = \frac{D_m^2 - D_n^2}{4(m-n)\lambda}$$

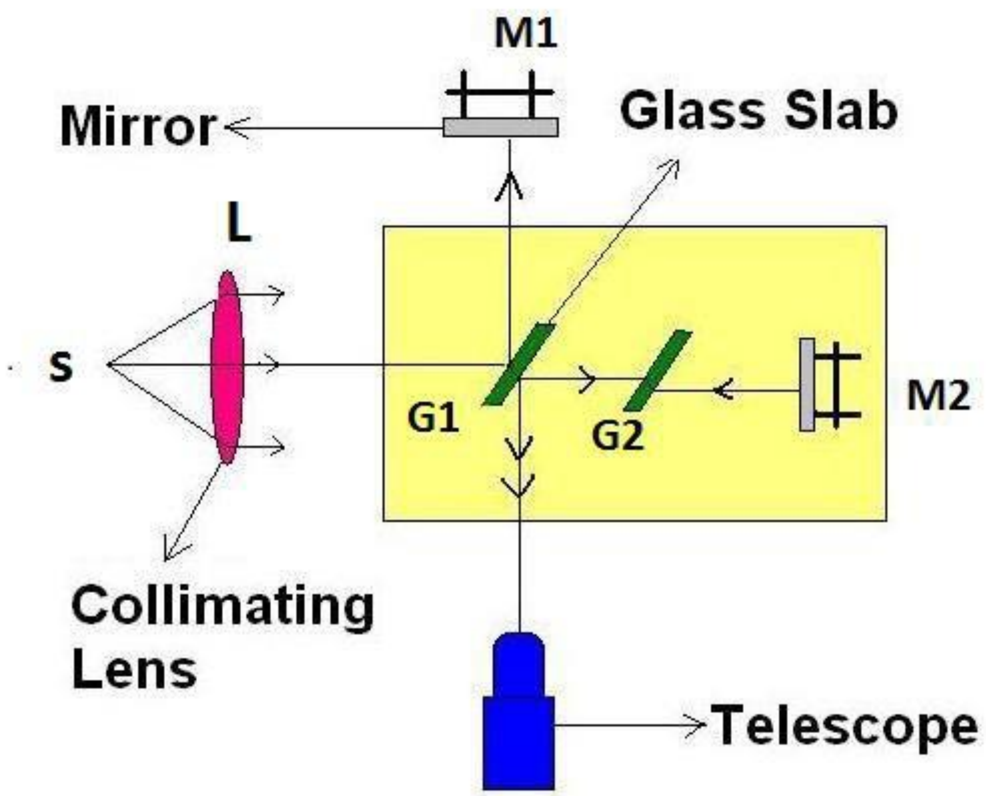
Fromthe graph, $\frac{AB}{CD} = \frac{D_m^2 - D_n^2}{(m-n)}$

TheradiusRofthePlanoconvex lenscanbeobtained withthehelp ofaspherometer.λ canbecalculatedbysubstitutingalltheabovevalues.

MICHELSONINTERFEROMETER

Construction:

- It consists of two excellent optically plane, highly polished plane mirrors $M_1$and M_2which are atrightangle to each other. There are two optically flatglass plates G_1 and $G_2$of same thickness and of the same material placed parallel to each other. These plates are inclined at an angle 45^0 with the mirrors M1 and M_2. T is a telescope which receives the reflected light from mirrors $M_1$and M_2.



Working:

- Monochromatic light from source S after being rendered parallel by a collimating lensL fallson the semi silvered glass plate G_1.
- It is divided into two parts, the reflected ray travels towards mirror M_1 and the transmitted ray travels towards mirror M_2 .
- The two rays fall normally on mirrors $M_1$and M_2respectively and are reflected back along their original paths.
- The reflected rayagain meets at the semisilvered surface of$G_1$and enters a short focus telescope T.
- The two rays entering the telescope are originally derived from the same single beam, hence they are in a position to produce interference fringes in the field of view of the telescope.
- A desired path difference can be introduced between the two reflected rays by moving the mirror M_1.
- It can be noted from figure that the reflected ray passes through G_1twice where as the transmitted ray does not do so even once.
- ThusintheabsenceofglassplateG_2thetwopathsarenot equal.
- To equalise the two paths, glass plate G_2ofsame thickness and materialas that of$G_1$is introduced in the path of transmitted ray.
- Becauseofthis nature,theglassplateG_2iscalledcompensating plate.
- The interference fringes may be straight, circular, parabolic etc depending upon path difference and angle between mirrors.

UsesofMichelson'sinterferometer:

Michelson'sinterferometer hasbeenusedforavarietyofpurposesfor example

1. Inthe determinationofwavelengthofmonochromatic source oflight.
2. Todeterminethedifference betweenthetwo neighbouringwavelengthsorresolutionof the spectral lines.
3. In the determination of refractive index and thickness of various thin transparent materials.

DIFFRACTION:

Introduction:

The bending of light around the edges of an obstacle is called diffraction. It was first observed by Gremaldy.

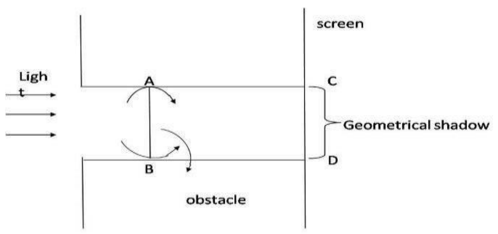


Fig: Diffraction

When light falls on an obstacle then the correspondinggeometricalshadowonthescreen should be completely dark. Practically the geometrical shadow consists of bright and dark fringes.These fringesareduetothesuperpositionofbended light wavesaroundthecornersof anobstacle. Theamountofbendingdependsuponthesizeofanobstacleandwavelengthof light.

When light falls on an obstacle whose size is comparable with the wavelength of light then light bends around the edges or corners of an obstacle and enters into the geometrical shadow. This bending of light is known as **diffraction.**

TypesofDiffraction:

Thediffractionphenomenaarebroadlyclassifiedintotwo types.

1. Fresnel'sDiffraction: _

Inthistypeofdiffraction, thesourceoflight andthescreenareplacedat finitedistance. In this, lenses are not necessary to study the diffraction. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

2. Fraunhofer'sDiffraction:

In this type of diffraction, the source and screen are placed at infinite distances. Herewe need lenses to studythe diffraction. This diffractioncanbe studied inanydirection. Inthis, the incident wavefront is plane.

DifferencebetweenFresneldiffractionandFraunhoferdiffraction:

S.No.	Fresneldiffraction	FraunhoferDiffraction
1	Sourceandscreenareplaced at finite distances.	Sourceandscreenareatplacedatinfinite distances.
2	Nolensesareused.	Lensesareused.
3.	Theincidentwavefrontisspherical or cylindrical.	Theincident wavefront isplanewavefront.
4.	The diffraction can be studied in the direction of propagation of light.	Thediffractioncanbestudiedinanyofpropagation of light.

Fraunhofer'sdiffractionatsingleslit:

Consider a slit AB ofwidth'e'. Let a plane wavefront WW'ofmonochromatic light of wavelengthλpropagatingnormallytowardstheslitisincidentonit.Thediffractedlight through the slit be focused by means of a convex lens on a screen placed in thefocal plane of the lens.

According to Huygens-Fresnel, everypoint onthe wavefront inthe plane ofthe slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit i.e., along the direction $OP_o$are brought to focus at $P_o$by the lens. Thus, P_o is a bright central image.

The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1on the screen. Depending on the path difference, the point P_1may have maximum or minimumintensities. Inordertofindout intensityatP_1,let usdrawaparallelAC fromAtothe light ray at B.

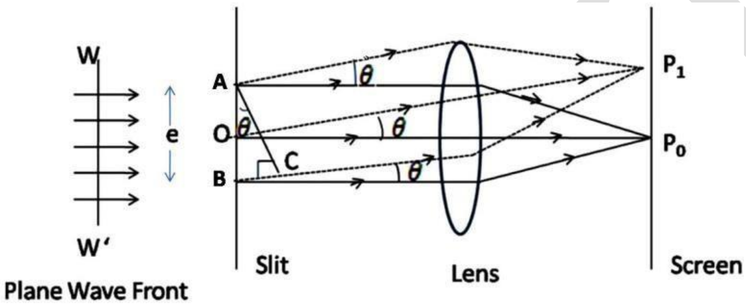


Fig: Fraunhofer Diffraction –Single Slit

ThepathdifferencebetweensecondarywaveletsfromAandBindirectionθ,is given by

$$\Delta = BC = AB\sin\theta = e\sin\theta$$

Therelationship betweenphasedifferenceandpathdifference isgivenby

$$Phasedifference = \frac{2\pi}{\lambda}\times pathdifference$$

$$= \frac{2\pi}{\lambda}\times e\sin\theta$$

Letthewidthoftheslit isdivided into ‘n’equalpartsandtheamplitudeofthewave from each part is ‘a’. Then the phase difference between any two successive waves is

$$\frac{1}{n}(Totalphase) = \frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = d \qquad (say)$$

Usingthemethodofvector additionofamplitudes theresultantamplitudeRisgivenby

$$R = \frac{a\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)} = \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

$$= \frac{a\sin\alpha}{\sin\left(\alpha/n\right)} \qquad \text{Where} \quad \alpha = \frac{\pi e\sin\theta}{\lambda}$$

$$= a\frac{\sin\alpha}{\alpha/n} \left(\because \frac{\alpha}{n} \text{ is very small}\right)$$

$$= \frac{na\sin\alpha}{\alpha}$$

Let $na = A$,

$$\boxed{R = \frac{A\sin\alpha}{\alpha}}$$

Resultant intensity(I)isproportionaltosquareofamplitude(R). Therefore,the intensity is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Intensitydistribution:

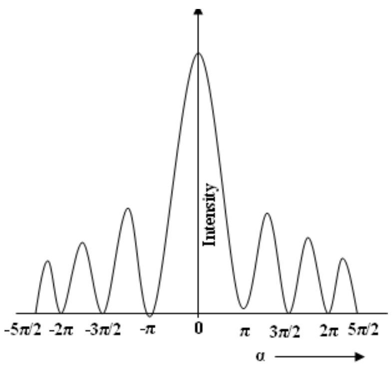


Fig: Intensity Distribution

The diffraction pattern consists of central principal maximum for α=0. There are subsidiary or secondary maxima of decreasingintensityoneithersidesofitatpositions$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$.Betweensecondarymaxima,thereareminimaat positions$\alpha = \pm\pi, \pm 2\pi$.

Diffractiongrating:

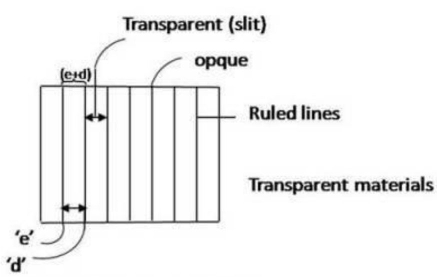


Fig: Diffraction grating

Diffraction grating is closed placed multiple slits. It consists of very large number of narrow slits side by side separatedbyopaquespaces.Theincident light istransmitted through the slits and blocked by opaque spaces. Such a grating is called transmission grating.

Whenlightpassesthroughthegrating,eachoneof the slit diffracts the waves. All the diffracted waves combine oneanotherproducingsharpandintensemaximaonthescreen.

A plane transmission grating is a plane sheet of transparent material on which opaque lines are made with a diamond point. The space between the rulings is equal and transparent and consists of parallel slits. The combined width of a ruling and a slitis called grating element.

Let ‘e’ be the width ofthe line and‘d’ be the width ofthe slit. Then (e + d) is knownas grating element. If ‘N’ is the no. of lines per inch on the grating then

$$N(e+d)=1''=2.54cm$$

$$e+d=\frac{2.54}{N}cm$$

Whenlight fallsonthegrating,the light getsdiffractedthrougheachslit.Asaresult, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as diffraction spectrum.

Gratingspectrum:

Theconditiontoformtheprincipalmaximainagratingisgivenby

$(e+d)\sin\theta=n\lambda$ iscalledgratingequation.

where(e+d) isthegratingelementandn=1, 2,…...

Instead of monochromatic source of light such as sodium vapour lamp, if white light source such as mercury is used then each diffracted order will have different colours atdifferent angles.

Forn=1

$$(e+d)\sin\theta_v=\lambda_v(for\,violet\,ray)$$

$$(e+d)\sin\theta_r=\lambda_r(for\,red\,ray)$$

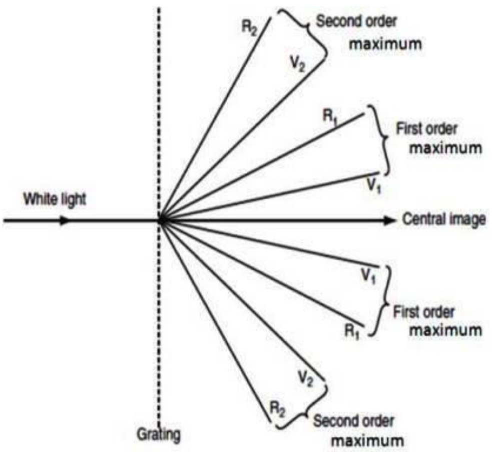


Fig: Grating spectrum for monochromatic source

Thusinthespectrumforagrating,there isno overlappingormixingofcoloursunlikethespectrum for a prism where different colours overlap. For a grating the angle of diffraction (i.e., angular dispersion) depends on λ and (e + d). Hence if two differentgratingsofsame(e+d)valuesarechosen, theywillproducesamedispersionandhencetheywillbeidentical.

Maximumno.ofordersavailablewithagrating:

Theprincipalmaximaina grating

$$(e+d)\sin\theta=n\lambda$$

$$\frac{1}{e+d}=N$$

Hence $r_1 < r_2,\ \mu_0 > \mu_e$

For $\theta = 90^0$, the maximum possible value of $\sin\theta$ is 1.

$$nN\lambda \le 1 \quad or\ n \le \frac{1}{N\lambda}$$

This gives the maximum number of orders possible and n is a integer.

Resolving power of a grating:

The resolving power of a grating represents its ability to form separate spectral lines for wavelengths very close together. It is measured by $\frac{\lambda}{d\lambda}$, where $d\lambda$ is the smallest wavelength difference that can just be resolved at wavelength λ.

Analysis:

Let a parallel beam of light of two wavelengths λ and $\lambda + d\lambda$ be incident normally on the grating. If the nth principal maximum of λ is formed in the direction θ_n, we will have $(e + d)\sin\theta_n = n\lambda$ where (e+d) is the grating element.

Now the grating equation for the minima is $N(e + d)\sin\theta = m\lambda$ where N is the total number of rulings on the grating and m can take all integral values except 0, N, 2N, …Nn, because these values of m give respective principal maxima. It is clear from figure that the first minimum of λ adjacent to nth principal maximum of $(\lambda + d\lambda)$ in the direction of increasing θ will be obtained for m = Nn+1. Therefore, for this minimum we have

$$(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda$$

$$(e+d)\sin(\theta_n + d\theta_n) = \left(\frac{nN+1}{N}\right)\lambda$$

According to Rayleigh criterion, the wavelengths λ and $\lambda + d\lambda$ are just resolved by the grating when the n^{th} maximum of $\lambda + d\lambda$ is also obtained in direction $\theta_n + d\theta_n$ i.e

$$(e+d)\sin(\theta n + d\,\theta n) = n(\lambda + d\lambda) \quad \text{..............} (4)$$

Figure shows the overlapping of principal maxima of two patterns

Comparing eqs (3) and (4), we get

$$\left(\frac{nN+1}{N}\right)\lambda = n(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

$$\lambda / d\lambda = nN$$

But $\left(\frac{\lambda}{d\lambda}\right)$ is the resolving power R of the grating. Therefore,

$$R = nN$$

$$R = \frac{(e+d)\sin\theta n}{\lambda}$$

As expected, the resolving power is zero for the central principal maximum (n=0), all wavelengths being indiffracted in this order.

UnitIII INTRODUCTIONTOSOLIDS

Freeelectrontheory:

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

Ithasbeendeveloped inthree mainstages:

1. Classicalfreeelectrontheory
2. QuantumFree ElectronTheory.
3. ZoneTheory.

- **Classical free electron theory**: The first theory was developed by Drude & Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.
- **Quantum Free Electron Theory**: In 1928 Sommerfield developed the quantum free electron theory. According to Sommerfield, thefree electrons move with a constant potential. This theory obeys quantum laws.
- **Zone Theory**: Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called "Band Theory of Solids". It gives complete informational study of electrons.

Classicalfreeelectrontheory:

Even though the classical free electron theory is the first theory developed to explain the electrical conduction of metals, it has many practical applications. The advantages and disadvantages of the classical free electron theory are as follows:

Advantages:

1. It explainstheelectricalconductivityand thermalconductivityofmetals.
2. Itverifiesohm'slaw.
3. Itisused to explaintheopticalpropertiesofmetals.
4. Metal composed of atoms in which electrons revolve around the nucleus are manystates available for occupation. Ifthe densityofstates is zero, no states canbe occupied at that energy level.
5. The valence electrons are freely moving about the whole volume ofthe metals like the molecules of perfect gas in a container
6. The free electrons moves in random directions and collide with either positive ions or other free electrons. Collision is independent of charges and is elastic in nature
7. Themovementsoffreeelectronsobeythelawsofclassicalkinetictheoryofgases
8. Potentialfieldremainsconstantthroughoutthelattice.
9. In metals, there are large numbers of free electrons moving freely within the metal i.e. thefreeelectronsorvalenceelectronsarefreetomoveinthemetallikegaseous

molecules, because nuclei occupy only 15% metal space and the remaining 85% space is available for the electrons to move.

Drawbacks:

1. Itfailstoexplaintheelectricspecificheatandthespecific heatcapacityofmetals.
2. Itfailstoexplainsuperconductingpropertiesofmetals.
3. Itfailstoexplainnewphenomenalikephotoelectriceffect,Comptoneffect,black–bodyradiation,etc.
4. ItfailstoexplainElectricalconductivity(perfectly)ofsemiconductorsorinsulators.
5. Theclassicalfreeelectronmodelpredictstheincorrecttemperaturedependenceof σ.Accordingtotheclassicalfreeelectrontheory,αT^{-1}.
6. Itfailstogiveacorrectmathematicalexpressionforthermalconductivity.
7. Ferromagnetismcouldn’tbeexplainedbythistheory.
8. Susceptibilityhasgreatertheoreticalvaluethantheexperimentalvalue.

Quantumfreeelectrontheoryofmetals:
Advantages:

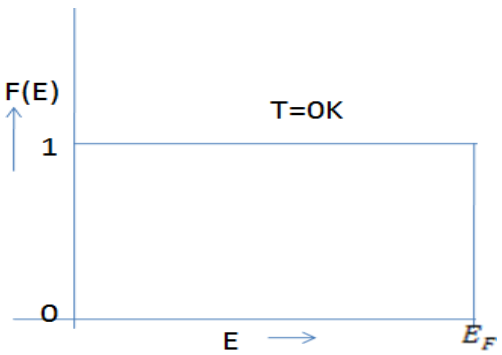
1. Alltheelectronsarenotpresentinthegroundstateat0K,butthedistributionobeys Pauli’sexclusionprinciple. At0K,thehighestenergylevelfillediscalledFermi-level.
2. Thepotentialremainsconstantthroughoutthelattice.
3. Collision of electrons with positive ion cores or other free electrons is independent ofcharges and is elastic in nature.
4. Energylevelsarediscrete.
5. Itwassuccessfultoexplainnotonlyconductivity,butalsothermionicemission paramagnetism, specific heat.

Drawbacks:

1. Itfailstoexplainclassificationofsolidsasconductors,semiconductorsandinsulators.

FermilevelandFermienergy:

The distribution of energy states in asemiconductor is explained by Fermi –Dirac statistics since it deals with the particles havinghalf integral spin like electrons. Consider that the assembly of electrons as electron gas which behaves like a system of Fermi particles or fermions. The Fermions obeying Fermi –Dirac statistics i.e., Pouli, sexclusion principle.



Fermienergy: Itistheenergyofstateatwhich the probabilityofelectronoccupationis½ at anytemperatureabove0K. Itseparatesfilledenergy states and unfilled energystates. The highest energylevelthat canbe occupied byanelectron at 0 K is called Fermi energy level

Fermilevel: Itisa levelat whichtheelectronprobabilityis½ at anytempabove0K(or) always it is 1 or 0 at 0K.

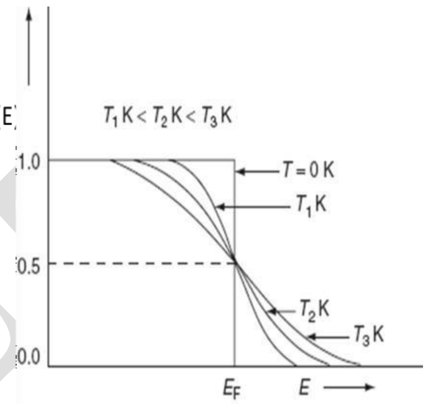
Therefore,theprobabilityfunctionF(E)ofanelectronoccupyinganenergylevelEisgivenby,

$$F(E)=\frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)} \quad \text{.................(1)}$$

Where $E_F$knownasFermienergyand itisconstantforasystem,

KistheBoltzmannconstantandTistheabsolutetemperature.

Case I : Probability of occupation at T= 0K,and $E<E_F$ThereforeF(E)=1,asperabove,clearlyindicatesthatatT=0 K,the energy level below the Fermi energy level $E_F$is fully occupied by electrons leaving the upper level vacant. Therefore,thereis100%probabilitythattheelectronsto occupy energy level below Fermi level.



CaseII: Probabilityofoccupationat T=0K,and$E>E_F$
Then

i.e., all levels below $E_F$are completely filled and al levels above $E_F$arecompletelyempty.AsthetemperaturerisesF (E).

CaseIII: ProbabilityofoccupationatT=0K,and $E=E_F$

$$(E) = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

Theaboveconditionstatesthat,T=0K, thereisa50%probabilityfortheelectronstooccupy Fermi energy.

The probability function F(E) lies between 0 and 1.

Hencetherearethreepossibleprobabilititiesnamely

F(E)=1 100%probabilitytooccupytheenergylevelbyelectrons.

F(E) =0 Noprobabilitytooccupytheenergylevelsbyelectronsandhence,it isempty. F(E) =0.5 50%probability of finding the electron in the energy level.

DensityofStates(DOS):

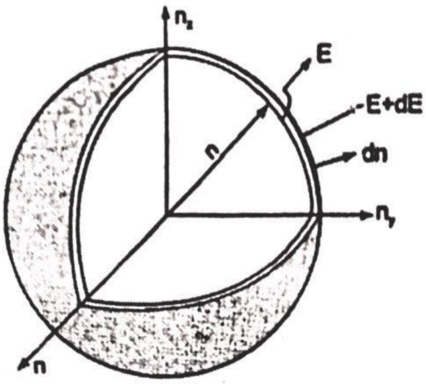
The number of electrons per unit volume in an energy level at a given temperature is equal to the product of density of states (number of energy levels per unit volume) and Fermi Dirac distribution function (the probability to find an electron).

$$n_c=\int g(E) \times f(E)dE \text{..........} \qquad (1)$$

where$n_c$istheconcentrationofelectrons, g(E) isthe densityofstates&F(E) isthe occupancy probability.

Thenumberofenergystateswitha particularenergyvalueE isdependingonhow many combinations of quantum numbers resulting in the same value n.

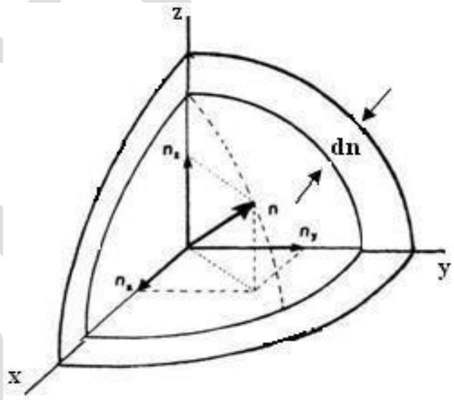
To calculate the number of energy states with all possible energies, we construct a sphere in 3D- space with 'n' as radius and every point (n_x, $n_y$and n_z) in the sphere represents an energy state.



As every integer represents one energy state, unit volume ofthis space contains exactlyone state .Hence;the number of states in any volume is equal to the volume expressedinunitsofcubesoflatticeparameters).Also $n^2 = n_x^2 + n_y^2 + n_z^2$

Consider a sphere ofradiusnand another sphere of radius n+dn with the energy values are E and (E+dE) respectively.

Therefore,thenumberofenergystatesavailablein the sphere of radius 'n' is byconsideringone octantofthe sphere (Here,thenumberofstatesinashellofthicknessdnata distance'n' incoordinatesystemformedby $n_{x,y} and n_z$andwilltakeonlypositivevalues,inthat sphere $\frac{1}{8}$ofthe volume willsatisfy this condition).



Thenumberofenergystateswithinasphereofradius (n+dn) is

$$\frac{1}{8}\left(\frac{4\pi}{3}\right)(n+dn)^3$$

ThusthenumberofenergystateshavingenergyvaluesbetweenE andE+dE isgivenby

$$g(E)dE = \frac{1}{8}\left(\frac{4\pi}{3}\right)(n+dn)^3 - \frac{1}{8}\left(\frac{4\pi}{3}\right)n^3$$

$$= \frac{1}{8}\left(\frac{4\pi}{3}\right)[(n+dn)^3 - n^3]$$

$$= \frac{\pi}{6}(3n^2 dn) = \frac{\pi}{2} n^2 dn$$

comparedto'dn',$dn^2 and dn^3 are very small$.
Neglecting higherpowersofdn

$$g(E)dE = \frac{\pi}{2} n^2 dn \text{........(2)}$$

Theexpressionforn^{th}energylevelcanbewrittenas ,

$$E = \frac{n^2 h^2}{8mL^2} \quad or, \quad n^2 = \frac{8mL^2 E}{h^2} \text{........(3)}$$

$$\Rightarrow n = \left(\frac{8mL^2 E}{h^2}\right)^{\frac{1}{2}} \text{........................ (4)}$$

Differentiatingeq.(3):

$$2n dn = \frac{8mL^2}{h^2} dE \quad => dn = \frac{1}{2n}\left(\frac{8mL^2}{h^2}\right) dE$$

∵bysubstituting1/nvalueindn,

$$n = \frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{\frac{1}{2}} \frac{dE}{E^{\frac{1}{2}}} \quad \text{.........(5)}$$

Substituten^{2}anddnfromeq.(3)and(5),weget

$$g(E)dE = \frac{\pi}{2}\left(\frac{8mL^2}{h^2}\right)dE \times \frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} \frac{dE}{E^{1/2}}$$

$$g(E)dE = \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE \quad \text{.........(6)}$$

AccordingtoPauli’sExclusionPrinciple,twoelectronsofoppositespincanoccupyeach energystate

Equation(6)should be multipliedby2

$$g(E)dE = 2 \times \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

After mathematicalsimplification,weget

$$g(E)dE = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}} E^{\frac{1}{2}} L^3 dE$$

Thedensityofenergystates g(E) dEperunitvolumeis givenby,

$$\boxed{g(E)dE = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE} \qquad \because L^3 = 1$$

BlochTheorem:

According to free electron model, a conduction electron in metal experiences constantpotential. But in real crystal, there exists a periodic arrangement of positively charged ions through which the electrons move. As a consequence, the potential experienced byelectrons is not constant but it varies with the periodicity of the lattice.In zone theory, as per Bloch, potential energy of electrons considered as varying potential with respect to lattice ‘a’.

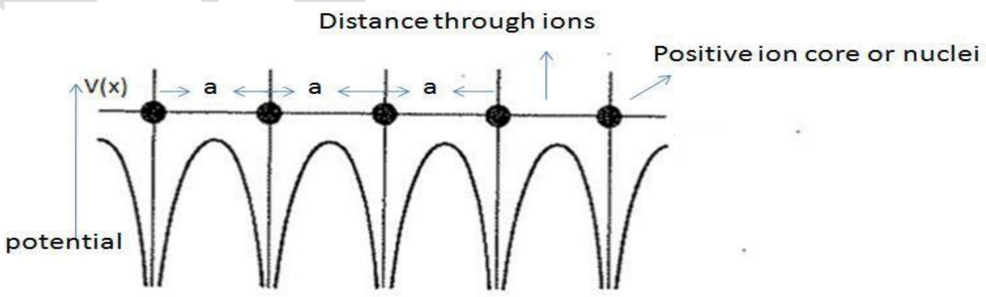


Fig:Variationofpotentialenergyinaperiodiclattice.

Let us examine one dimensional lattice as shown in figure. It consists of array of ionic cores along X-axis. A plot of potentialV as a function of its position is shown in figure.

Fromgraph:

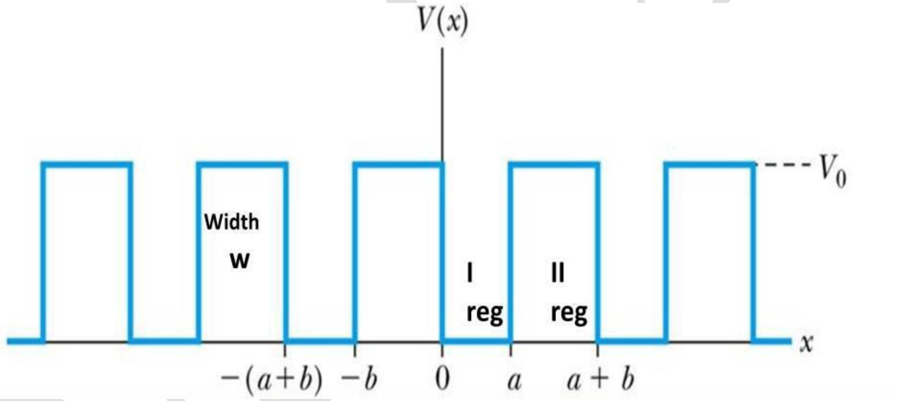
Atnucleiorpositiveioncores,thepotentialenergyofelectronisminimumandin-between nuclei,theP.E.isconsideredasmaximumw.r.to.Latticeconstant'a'.
ThisperiodicpotentialV(x)changeswiththehelpoflatticeconstanta,$V(x)=V(x+a)$
('a'isthe periodicityofthelattice)
Tosolve,byconsidering Schrodinger'stimeindependentwaveequationinonedimension,

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}[E-(x)]\psi=0......(1)$$

Bloch's1DsolutionforSchrodingerwaveequation(1) $\psi_k(x)=u_k(x)exp(ikx)$ .... (2)
where$u(x)=u_k(x+a)$
Here$u_k(x)$ -periodicityofcrystallattice,modulatingfunction,k-propagationvector= $\frac{2\pi}{\lambda}$ e^{ikx} isplanewave.
By applyingeq.n (2)toeq.n (1),itisnoteasy tosolve Schrodingerwave equation andBloch cannot explains complete physical information about an e^-in periodic potential field. Then Kronig Penny model was adopted to explains the electrical properties of an e^-.

Kronig-Penneymodel:

Kronig –penny approximated the potentials of an e^-s inside the crystal in terms of the shapesof rectangular steps as shown, i.e. square wells is known as Kronig Penny model.
i.e. The periodic potential is taken in the form of rectangular one dimensional array of square well potentials and it is the best suited to solve Schrodinger wave equation.



It is assumed that the potential energy is zero when x lies between 0 and a, and is considered as I region. Potential energy is V_0, when x lies between –b and 0.And considered as II region.

Boundaryconditions:
$(x)=0$,$where\ xliesbetween0<x<a–Iregion$ $V(x)=V_0$,$herexliesbetween-b<x<0-IIregion$
This model explains many of the characteristic features of the behavior of electrons in a periodic lattice.
The wave functionrelated to this model maybe obtained bysolving Schrodinger equations for the two regions,

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}E\psi=0,for0<x<a with V(x)=0 .... (1)$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0, \text{for} -b < x < 0 \text{ with } V(x) = V_0 \text{………(2)}$$

Again,

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \text{……..(3)} \qquad \text{where } \alpha^2 = \frac{2mE}{\hbar^2} \text{ and } \alpha = \frac{2\Pi}{h}\sqrt{2mE}$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \text{…….(4)} \qquad \text{where } \beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

ThesolutionoftheseequationsfromBlochtheorem,$_k(x) = u_k(x)exp(ikx)$. From figure, square well potentials, ifV_0increases, the width of barrier 'w' decreases, if V_0decreases the width of barrier w increases. But the (product) barrier strength $V_0 w$ remains constant.

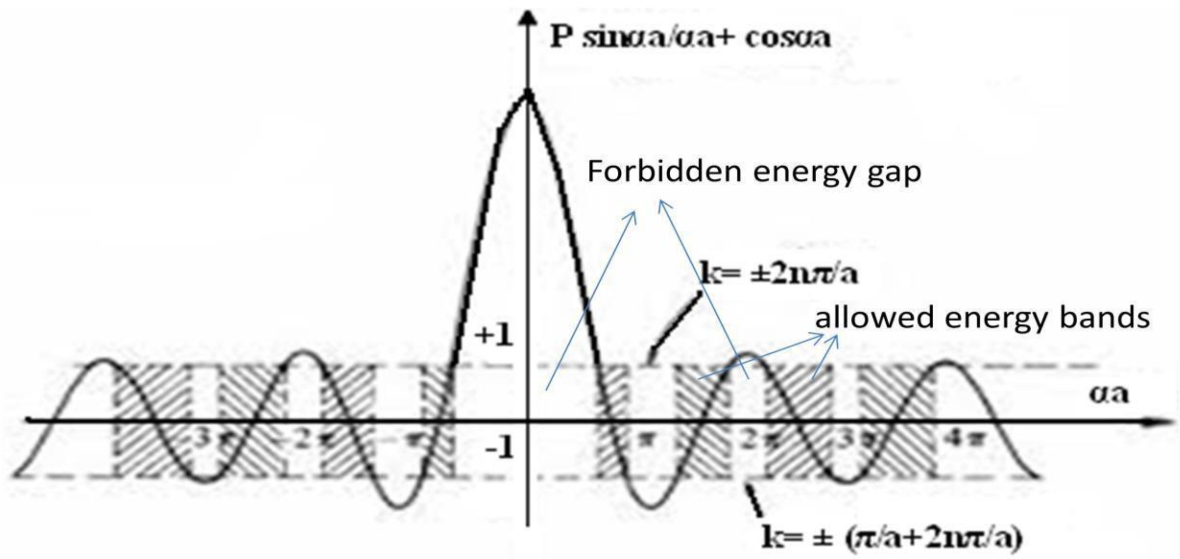
Togetthis,differentiatingaboveSchrodingerwaveequations3&4w.r.tox,andbyapplying boundary conditions of x (w.r.to their correspondingΨ), to known the values of constants A, Bofregion-I,C,D-forreg-II,wegetmathematicalexpression(bysimplification)

$$\boxed{coska = P\frac{sin\alpha a}{\alpha a} + cos\alpha a}$$

where,

$$P = \frac{4\pi^2 ma}{h^2} V_0 w \qquad \text{and} \qquad \alpha = \frac{2\pi}{h}\sqrt{2mE}$$

P-varyingterm,knownasscatteringpower.



And'v_0b' isknownasbarrierstrength.

Conclusions:

1. TheL.H.Sisacosinetermwhichvariesbetweenthelimits-1and+1,andhencethe R.H.Salsovariesbetweentheselimits.Itmeansenergyisrestrictedwithin-1to+1 only.

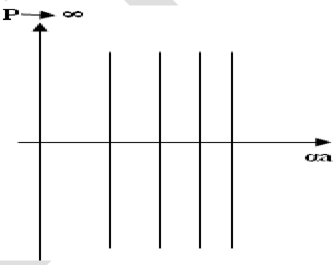
2. Ifthe energy ofe^-lies between -1 to +1, are called **allowed energy bands** and it is shownbyshadedportioninenergyspectrum. This meansthat 'αa'cantakeonlycertain range of values belonging to allowed energy band.
3. Asthevalueofαaincreases,thewidthoftheallowed energybandsalsoincreases.
4. Ifenergy ofe^-s notliesbetween -1 to+1 are known as **forbiddenenergybands** and it is decreases w.r.to increment of αa.
5. Thus, motion ofe^-s. in a periodic lattice is characterized by the bands of allowed & forbidden energy levels.

Case1:

1. P→∞

   IfP→∞,theallowedbandreducestoasingle(line)energylevel,givesussteeper lines.

Wehave



$cos ka(\frac{\alpha a}{p})=\sin\alpha a+cos\alpha a(\frac{\alpha a}{p})$

$P\rightarrow\infty, \frac{1}{\infty}=0 \quad then \quad sin\alpha a=0$

$sin\alpha a=\sin n\pi$

$\alpha a=n\pi$

$\alpha^2a^2=n^2\pi^2$

$\alpha^2=\frac{n^2\pi^2}{a^2}$

$\frac{2mE}{\hbar^2}=\frac{n^2\pi^2}{a^2}$

$E=\frac{n^2\pi^2\hbar^2}{2ma^2}=\frac{n^2\pi^2\text{h}^2}{2ma^24\pi^2}$

$E=\frac{n^2\text{h}^2}{8ma^2}$ $,here a is lattice\ constant$

Itmeans,it(zonetheory)supportsquantumfreeelectrontheory.

Case2:

P→0,Wehave

$cos ka=P\frac{sin\alpha a}{\alpha a}+cos\alpha a$

$\alpha a=ka$

$\alpha=k$

$\alpha^2=k^2$

$\frac{2mE}{\hbar^2}=(\frac{2\pi}{\lambda})^2=\frac{4\pi^2}{\lambda^2}$

$E=\frac{4\pi^2\hbar^2}{2m\lambda^2}$

$E=\frac{4\pi^2\text{h}^2}{2m\lambda^24\pi^2}$

$E = \frac{1}{2} m v^2$

It gives us kinetic energy of an electron. It means zone theory supports classical free electron theory at this situation electron completely free electron not bounded with allowed and forbidden gaps (and no energy level exists).

ThusbyvaryingPfrom0to∞,wefindthatthecompletelyfreeelectron(s)becomes completelyboundtoBrillouinZone**.**

BrillouinZoneORE-Kdiagram:

TheBrillouinzonearetheboundariesthat aremarkedbythe valuesofwavevectork,inwhich electrons can have allowed energy values. These represent the allowed values of k of the electrons in 1D, 2D,&3D.

Wehave,theenergyoftheelectroninaconstantpotentialboxis,

$$E = \frac{n^2 h^2}{8ma^2} \text{....(1)} \quad \text{wherea=lengthofthebox.}$$

But,

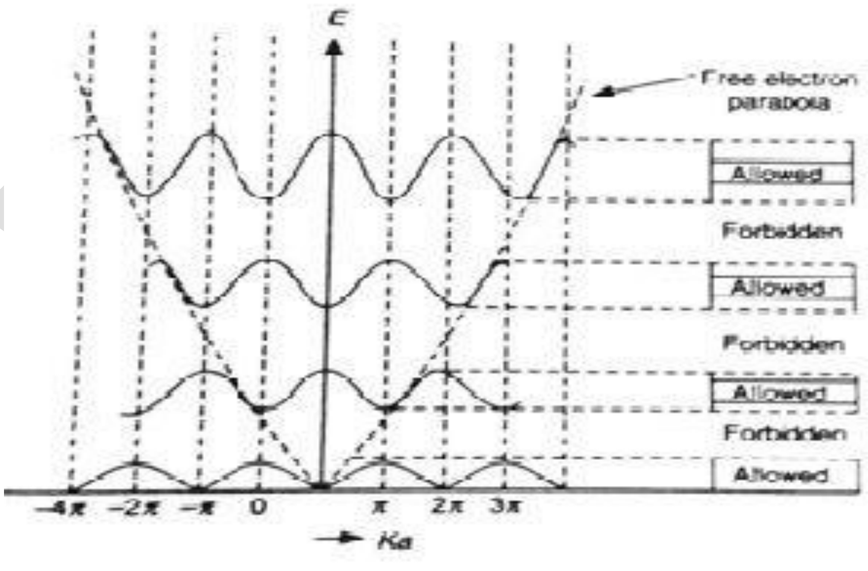
$$k = \frac{n\pi}{a} \Rightarrow k^2 = \frac{n^2 \Pi^2}{a^2}$$

$$\frac{n^2}{a^2} = \frac{k^2}{\pi^2} \quad \text{.........(2)}$$

Substituteeqn(2)in(1),weget$E = \frac{k^2 h^2}{8m\pi^2}$;$\propto k^2$.Itrepresentsparabolicequation.

Agraph isdrawnbetweenthetotalenergy(E) and thewavevector k,forvariousvaluesofk.

i.e.$k = \frac{n\pi}{a}$;=$\pm 1, \pm 2, \pm 3$,......



Itistheenergyspectrumofanelectronmoving inpresenceofaperiodicpotentialfieldand is divided into allowed energy regions (allowed zones) or forbidden energy gaps (forbidden zones).

Allowedenergyvalueslieintheregionk=-π/ato=+π/a.Thiszoneiscalledthefirst Brillouinzone.Afterabreakintheenergyvalues,calledforbiddenenergyband,wehave anotherallowedzonespreadfromk=-π/ato-2π/aand+π/ato+2π/a.Thiszoneiscalledthe secondBrillouinzone.Similarly,higherBrillouinzonesareformed

Conceptofeffectivemassof electron:

When an electron in a periodic potential of lattice is accelerated by an known electric field or magnetic field, thenthe mass ofthe electronis called effective mass and is represented by $m*$. To explain, let us consider an electron of charge 'e' and mass 'm' moving inside a crystal lattice of electric field E.

Thenbytaking knownexpressionF=ma,can beconsideredhereas$F=m*a$..........................(1)

Theacceleration$a=\frac{eE}{m}$ isnotconstantintheperiodiclatticebutvariesduetothechangein electronicmass.

Iffreeelectronunderwavepacket,thegroupvelocity$V_g$correspondingtotheparticle's velocitycanbewrittenas

$$v_g=\frac{d\omega}{dk}=2\pi\frac{dv}{dk}=\frac{2\pi dE}{hdk}=\frac{1}{\hbar}\frac{dE}{dk}\text{... ....(2)} \qquad where E=hv$$

Therateofchangeofvelocity isknown as

$$Acceleration, a=\frac{dv_g}{dt}=\frac{1}{\hbar}\frac{d^2E}{dkdt}$$

$$a=\frac{1}{\hbar}\frac{dE}{dt}\frac{dk}{dk}=\frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}\text{.....(3)}$$

$$From quantum mechanics relation, =\hbar k\text{...(4)}$$

$$and F=\frac{dp}{dt}\text{.......(5)}$$

$By\ differentiating eq$ (4)w.r.tot,andbysubstituting eq(5)

$$\hbar\frac{dk}{dt}=\frac{dp}{dt}=F\Rightarrow\frac{dk}{dt}=\frac{F}{\hbar}\text{ ....(6)}$$

bysubstitutingeq.(6) ineq (3),

$$\therefore a=\frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{F}{\hbar}$$

Byrearrangingtheabovetermandbycomparingwitheq.(1)

$$F=\left[\frac{\hbar^2}{\left(d^2E/dk^2\right)}\right]a$$

$$\therefore m^*=\frac{\hbar^2}{\left(d^2E/dk^2\right)}$$

Isknownasexpressionform^*anditisdependsonEandK.

OriginofenergybandformationinSolids:

Thebandtheoryofsolidsexplainstheformationofenergybandsanddetermines whether a solid is a conductor, semiconductor or insulator.

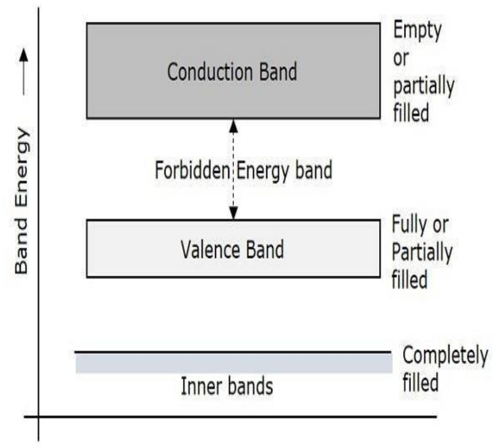
The existence of continuous bands of allowed energies can be understood starting with the atomic scale. The electrons of a single isolated atom occupy atomic orbitals, which form a discrete set of energy levels.

When two identical atoms are brought closer, the outermost orbits of these atoms overlap and interact. When the wave functions of the electrons of different atoms begin to overlap considerably, the energy levels corresponding to those wave functions split.

If more atoms are brought together more levels are formed and for a solid of N atoms, each of the energy levels of an atom splits into N energy levels. These energy levels are so close that they form an almost continuous band. The width of the band depends upon thedegree of overlap of electrons of adjacent atoms and is largest for the outermost atomic electrons.

As a result ofthe finite widthofthe energybands, gaps areessentiallyleftover between the bands called forbidden energy gap.

The electrons first occupy the lower energy levels (and are of no importance) then theelectrons in the higher energy levels are of important to explain electrical properties of solidsand these are called valence band and conduction band.



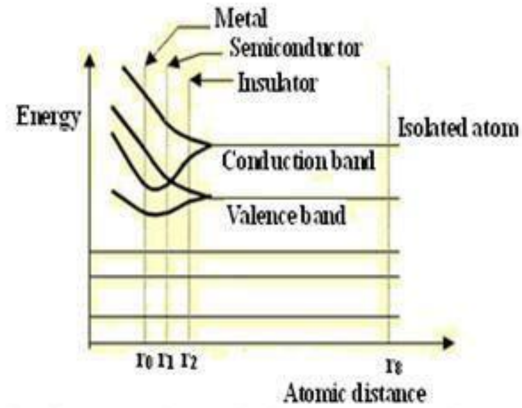


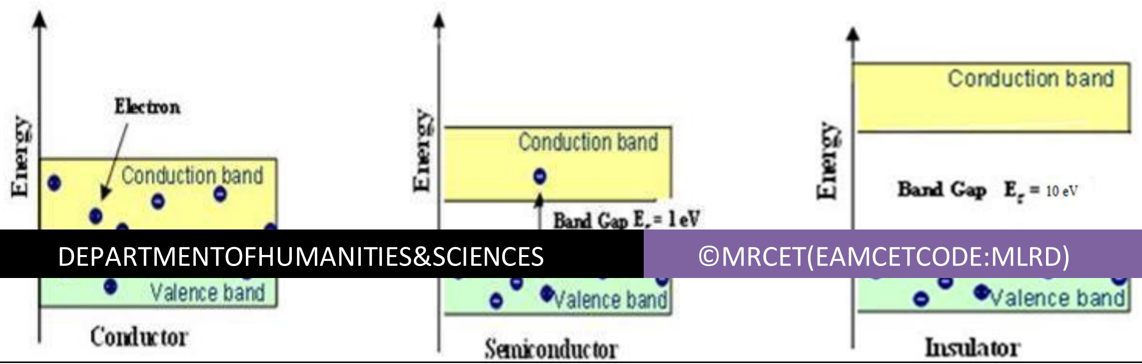
Fig: Formation of energy bands when atoms are closer

Valence band: Aband occupied by valence electrons and is responsible for electrical, thermal and optical properties of solids and it is filled at 0K.

Conduction band: A band corresponding to outer most orbit is called conduction band and is the highest energy band and it is completely empty at 0K.
The forbidden energygap between valence band conduction band is knownas the energy band gap. By this solids are classified in to conductors, semiconductors and insulators.

Classificationofsolidsintoconductors,Semiconductors&Insulators:

Basedontheenergybanddiagrammaterialsorsolidsareclassifiedasfollows:



Conductors: Inthis kind of materials, there is no forbiddengap betweenthe valence band and conduction band. It is observed that the valence band overlaps with the conduction band in metals as shown in figure. There are sufficient numbers of free electrons, available forelectrical conduction and due to the overlapping of the two bands there is an easy transition of electrons from one band to another band takes place, and there no chance for the presence of holes. Resistivityof conductors is very small and it is very few milli ohm meters.(Ω m).

Examples:Allmetals(Na,Mg,Al,Cu,NiCu,Ag,Li,Aretc)

Semiconductors:

In semiconductors, there is a band gap exists between the valence band and conduction band and it is very less and it is the order of -1 to 2 eV are known as semiconductors. It will conduct electricitypartiallyat normalconditions. The electricalresistivity valuesare 0.5to 10^3ohm meter. Due to thermal vibrations within the solid, some electrons gain enough energy to overcome the band gap (or barrier) and behave as conduction electrons. Conductivity exists here due to electronics and holes.

Examples:Silicon,Germanium,GaAs.

Insulators:

In insulators, the width of forbidden energy gap between the valence band andconduction band is very large. Due to large energy gap, electrons cannot jump from V.B to C.B.Energy gapis of the order of~10 eV and higher than semiconductors. Resistivity values of insulators are$10^7$to 10^{12}ohm-m. Electrons are tightly bound to the nucleus, no valence electrons are available.

Examples: Wood,rubber,glass.

1) UNIT -4

2) LASER AND FIBER OPTICS

3) Explaini)Metastablestateii)opticalpumpingiii)population inversion

Metastable state: The excited state, which has a long life time, is known as metastable state.
Optical pumping: Thisprocess is required to achieve population inversion and used inRuby laser.
Pumping processisdefinedas:“Theprocesswhichexcitestheatomsfromgroundstateto excited state to achieve population inversion”.
PopulationInversion:
Generally, number of atoms in the ground state is greater than the number of atoms in higher energy states.
But in order to produce a laser beam, the minimum requirement is stimulated emission.
Stimulatedemissiontakesplaceonlyifthenumberofatomsinthehigherenergylevelisgreater than the number of atoms in the lower energy level.
Simply population inversion is nothing but number of atoms in higher energy level is greater than the number of atom in lower energy level.

4) Definespontaneousandstimulatedemissionofradiation?

SpontaneousEmission: Whenanatomintheexcitedstateemitsaphotonofenergy‘hv’ comingdownto groundstatebyitselfwithoutanyexternalagency,suchan emissioniscalled spontaneous emission.Atom* □atom + hv.
Photonsreleasedinspontaneousemissionarenotcoherent.Hencespontaneousemissionisnot useful for producing lasers.
Stimulated Emission: When an atom in the excited state, emits two photons of same energy ‘hv’ while coming to down to ground state with the influence of an external agency, such an emissioniscalledstimulatedemission.Atom*□atom+2hv.

- In the two photons one photon induces the stimulated emission and the second one is released by the transition of atom from higher energy level to lower energy level.
- Both the photons are strictly coherent. Hence stimulated emission is responsible forlaser production.

5) Explainthebasicprinciple ofoptical fiber?

- Optical fibers are the waveguides through which electromagnetic waves of optical frequency range can be guided through them to travel long distances.
- Anopticalfiberworksontheprincipleoftotalinternalreflection(TIR).
- **Total Internal Reflection**: when a ray of light travels from a denser medium into a rarer medium and if the angle of incidence is greater than the critical angle then the light gets totally reflected into the denser medium

6) Explaini)NumericalApertureii)Acceptanceangle

i) NumericalAperture:
Numericalapertureof afiberis ameasureof itslightgatheringpower.
“TheNumericalAperture(NA)isdefinedasthesineofthemaximumacceptanceangle” The light gathering ability of optical fiber depends on two factors i.e.,(I)Core diameter(ii)NA

NAisdefinedassineoftheacceptanceangle i.e.,

$NA=\sin\theta_A$ i.e $NA=\sqrt{n_1^2-n_2^2}$

Theefficiencyofopticalfiberisexpressedintermsof NA;itiscalledasfigureofmeritof optical fiber.

ii) AcceptanceAngle:

Allrightrays fallingonopticalfiberarenottransmittedthroughthefiber. Onlythoselightraysmaking$\theta_i>\theta c$atthecore-claddinginterfacearetransmittedthroughthe fiberbyundergoingTIR.Forwhichangleofincidence,therefractionangleisgreaterthan90^0 will be propagated through TIR.

There by Acceptance Angle is defined as: The maximum angle of incidence to the axis of opticalfiberatwhichthelightraymayenterthefibersothatitcanbepropagatedthroughTIR.

5. Whatarethe main sectionsof opticalfiber?Describethestepindexopticalfiber?

- Anoptical fiber consists of three(3) co-axial regions.
- Theinnermostregionisthelight-guidingregionknownas"Core".Itissurroundedby a middle co-axial regional known as "cladding". The outer most regions which completely covers the core & cladding regions is called "sheath or buffer jacket".
- Sheath protects the core & cladding regions from external contaminations, in addition to providing mechanical strength to the fiber.
- Therefractiveindexofcore(n_1)isalwaysgreaterthantherefractiveindexofcladding (n_2) i.e., $n_1> n_2$to observe the light propagation structure of optical fiber.

StepIndexoptical fiber:

➢ Basedonvariationinthecorerefractiveindex(n_1),opticalfibersaredividedintotwotypes

1. Stepindexfiber
2. Gradedindexfiber

➢ Stepindexfibers haveboth single&multimodepropagations.

6) Writeashortnoteonattenuationin optical fibers.

Usually,thepoweroflightattheoutputendofopticalfiberisless thanthepowerlaunched at the input end, then the signal is said to be attenuated.

Attenuation: Itistheratioofinputopticalpower(P_i)intothefibertothepoweroflight coming out at the output end (P_o).

Attenuationcoefficientisgivenas,$\alpha=10/L\log_{10}P_i/P_o$db/km.

Attenuation is mainly due to

1. Absorption.
2. Scattering.
3. Bending.

7) **Write down advantages of fiber optics in communication system Or What are the Advantages of optical fibers over metallic cables**?

- Optical fibers allow light signals of frequencies over a wide range and hence greater volumeofinformationcanbetransmittedeitherindigitalformorinanalogformwithin a short time.

- Inmetalliccablesonly48conversationscanbemadeatoncewithoutcrosstalkswhere as in optical fibers more than 15000 conversations can be made at once without cross talks.
- Light cannot enter through the surface of the optical fiber except at the entryinterface i.e.,interferenceb/wdifferentcommunicationchannelsisabsent.Hencepurityoflight signal is protected.
- Optical signal do not produce sparks like electrical signals and hence it is safe to use optical fibers.
- External disturbances from TV or Radio Stations power electronic systems and lightening cannot damage the signals as in case of metallic cables.
- Materials used in the manufacture of optical fibers areSiO_2, plastic, glasses which are cheaper & available in plenty.

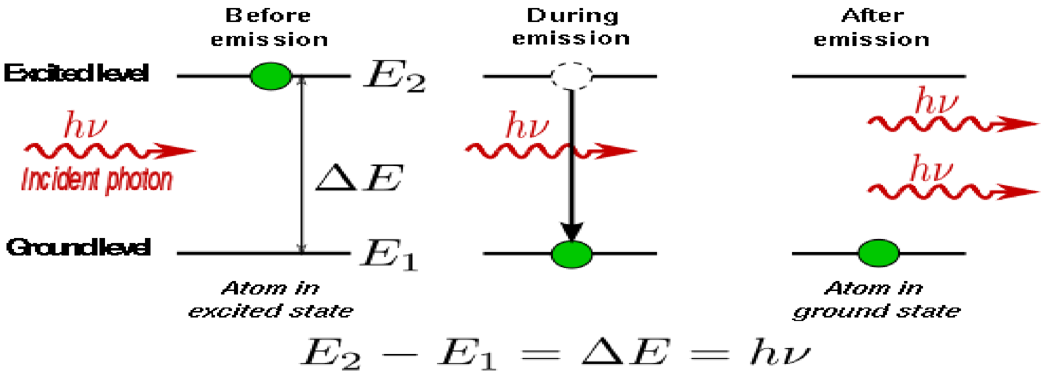
Part-B(Descriptive-10marks)

1) WhatisAcronymof aLaser,absorption,spontaneousandstimulatedemissions?

- **Laser:**Lasermeans**LightAmplificationbyStimulatedEmissionofRadiation.**
- **Absorption:**Whenatatomabsorbsanamountofenergy‘hv’intheformofphoton from the external agency and excited into the higher energy levels from ground state, thenthisprocessisknownasabsorption.Atom+hv atom*
- **Spontaneous Emission:** When an atom in the excited state emits a photon of energy ‘hv’ coming down to ground state by itself without any external agency, such an emissioniscalledspontaneousemission.Atom* atom+hv
- Photons released in spontaneous emission are not coherent. Hence spontaneous emission is not useful for producing lasers.
- **Stimulated Emission:** When an atom in the excited state, emits two photons of same energy‘hv’whilecomingtodowntogroundstatewiththeinfluenceofanexternal agency,suchanemissioniscalledstimulatedemission.Atom* atom+2hv
- In the two photons one photon induces the stimulated emission and the second one is released by the transition of atom from higher energy level to lower energy level.
- Both the photons are strictly coherent. Hence stimulated emission is responsible for laser production.

2) Explainprinciple oflaser/lasing action?

- **LaserProduction Principle:**
- Two coherent photons produced in the stimulated emission, interacts with other twoexcited atoms, resulting in four coherent photons.
- Thus, coherent photons are multiplied in a lasing medium. The continuous successive emission of photons results for the production of laser beam.

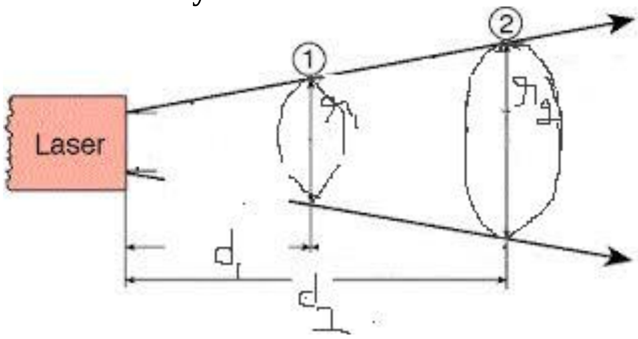


3) Whatarethecharacteristics/strikingfeatures/PropertiesofLaserLight?

CharacteristicsofLaserBeam: Someofthespecialpropertieswhichdistinguishlasersfrom ordinary light sources are characterized by:

1. Directionality
2. HighIntensity
3. Mono-chromacity
4. Coherence

1.Directionality:



Laser emits radiation only in one direction. The directionality of laser beam is expressed in terms of angle of divergence (Φ)

DivergenceorAngular Spreadisgivenby**Φ= r_2-r_1/d_2-d_1**

Whered_1,$d_2$areanytwodistancesfromthelasersourceemittedandr_1,$r_2$aretheradiiofbeam spots at a distance $d_1$and $d_2$respectively as shown in above figure. Laser light having less divergence, it means that laser light having more directionality.

2. HighIntensity: Generally,lightfromconventionalsourcespreaduniformlyinalldirections. For example, take 100 watt bulb and look at a distance of 30 cm, the power enter into the eye is less than thousand of a watt. This is due to uniform distribution of light in all directions. But in case of lasers, light is a narrow beam and its energy is concentrated within the smallregion. The concentration of energy accounts for greater intensity of lasers.

3. Monochromacity: Thelightemittedbylaserishighlymonochromaticthananyoftheother conventional monochromatic light. A comparison b/w normal light and laser beam, ordinary sodium(Na)lightemitsradiationatwavelengthof5893A^0withthelinewidthof1A^0.ButHe- Ne laser of wave length 6328A^0 with a narrow width of only10^{-7} A^0 i.e., Monochromacityof laser is 10 million times better than normal light.

ThedegreeofMonochromacityofthelightisestimatedbylineofwidth(spreadingfrequency of line).

4. Coherence: If any wave appears as pure sine wave for longtime and infinite space, then it is said to be perfectly coherent.

Practically,nowaveisperfectlycoherentincludinglasers.Butcomparedtootherlightsources, lasers have high degree of coherence because all the energy is concentrated within the small region. There are two independent concepts of coherence.

i) Temporalcoherence(criteriaoftime)

ii) Spatialcoherence(criteriaofspace)

4) Explaintheconceptofpopulation inversionandpumping in lasers?

PopulationInversion:

- Generally, number of atoms in the ground state is greater than the number of atoms in higher energy states.
- Butinordertoproducealaserbeam,theminimumrequirementisstimulatedemission.
- Stimulatedemissiontakesplaceonlyifthenumberofatomsinthehigherenergylevel is greater than the number of atoms in the lower energy level.
- Simply population inversion is nothing but number of atoms in higher energy level is greater than the number of atom in lower energy level.
- So,ifthereisapopulationinversiontherebyonly stimulatedemissionwillableto produce laser beam.
- Populationinversionisassociatedwiththree Phenomenon.
    - Stimulatedemission
    - Amplification
    - PumpingProcess
- Stimulated Emission: If majority of atoms are present in higher energy state than the process becomes very easy.
- Amplification: If 'N', represents number of atoms in the ground state and 'N_2' represents number of atoms in the excited state than the amplification of light takes place only when $N_2 > N_1$.
- If $N_2 > N_1$,therewillbeapopulationinversionsoinducedbeamandinducedemission are in the same directions and strictly coherent than the resultant laser is said to be amplified.
- Boltzmann's principle gives the information about the fraction of atom found on average in any particulars energy state at equilibrium temperature as
- $\frac{N1}{N2}$=exp(E_2-E_1/KT)=exp(ΔE/KT)

  $\frac{N1}{N2}$=exp(hv/KT)

PumpingProcess:

- Thisprocessisrequiredtoachievepopulation inversion.
- Pumpingprocessisdefinedas:"Theprocesswhichexcitestheatomsfromgroundstate to excited state to achieve population inversion".
- Pumpingcanbedone bynumberofways
- i) Optical Pumping excitation bystrongsourceof light (flashingofa Camera)
- ii) ElectricalPumping excitationbyelectron impact
- iii) ChemicalPumping excitationbychemicalreactions
- iv) DirectConversion Electricalenergyis directlyconvertedinto

radiantEnergyindeviceslikeLED's,population Inversion is achieved in forward bias.

5) WhatareEinstein'scoefficientsandexplaintherelationamong them?

or

Derivetherelationbetweentheprobabilitiesofspontaneousemissionandstimulated emission in terms of Einstein's coefficient?

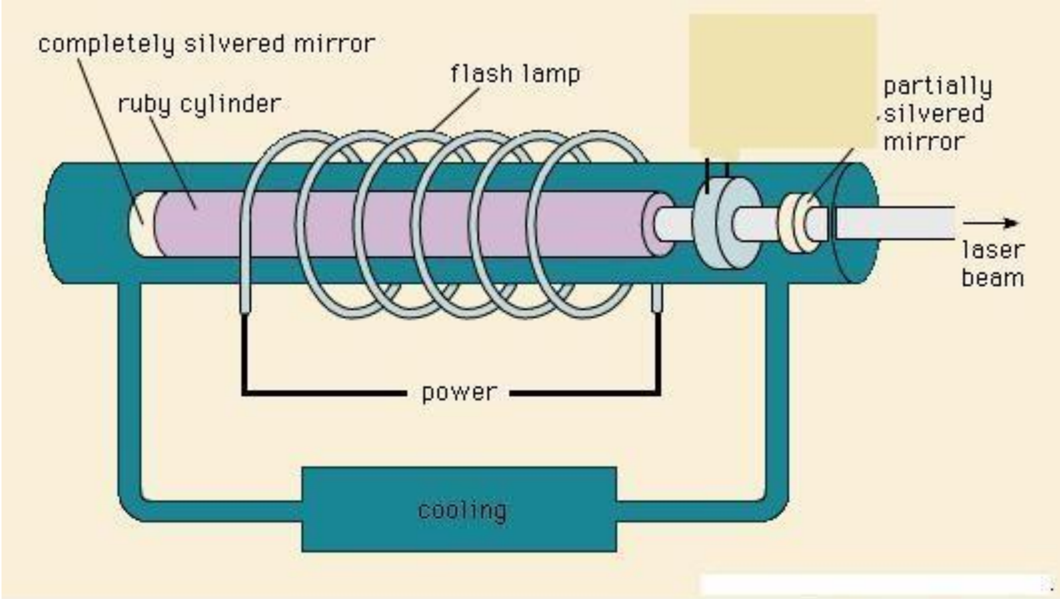
Einstein'sTheoryofRadiation:

- In 1917, Einstein predicted the existence of two different kinds of processes by which an atom emits radiation.
- Transitionb/wtheatomicenergystatesisstatisticalprocess.Itisnotpossibletopredict which particular atom will make a transition from one state to another state at a particular instant. For an assembly of very large number of atoms it is possible to calculate the rate of transitions b/w two states.
- Einsteinwasthefirsttocalculatetheprobabilityofsuchtransition,assumingtheatomic system to be in equilibrium with electromagnetic radiation.
- The number of atoms excited during absorption in the time'Δt' is given by: N_{ab}= Q $N_1B_{12}\Delta t$, Where N_1= number of atoms in state 'E_1', Q = Energy density of induced beam and B_{12}= Probability of an absorption transition coefficient.
- The number of spontaneous transitions N_{sp}taking place in time 'Δt' depends on only no. of atoms N_2lying in excited state. N_{sp}= $A_{21}N_2\Delta t$, Where A_{21}= probability of spontaneous transition.
- The number of stimulated transitions N_{st}occurring during the time Δt may be written as: N_{st}= $B_{21}N_2\Delta t$, Where B_{21}= probability of stimulated emission.
- Under the thermal equilibrium number of upward transitions = number of downward transitions per unit volume per second.
- So,wecanwrite:$A_{21}N_2+B_{21}N_2Q=B_{12}N_1Q$1
- $Q=A_{21}N_2/B_{12}N_1-B_{21}N_2$ ------------------------ >2
- DividingbyB$_{21}$N$_2$in allterms, $Q =(A_{21}/B_{21})$ x1/$(B_{12}N_1/ B_{21}N_2)$– 1----------------- >3
- BysubstitutingN$_1$/N$_2$=exp (hυ/kT) from Boltzmann Distribution law,
- $Q=(A_{21}/ B_{21})1/(B_{12}/ B_{21})\exp (h\upsilon/kT)– 1$----------------- >4
- Aboveequationmustagreewithplanksenergydistribution –radiationformula. Q = $\hbar\upsilon^3/\prod^2C^3$ 1/exp (hυ/kT) -1 ------------------------------> 5
- Fromequations 4&5,$B_{12}=B_{21}$, wegetA$_{21}$/B$_{21}$=$\hbar\upsilon^3/\prod^2C^3$
- Theco-efficientsA_{21}, B_{12},B_{21}areknownasEinstein coefficients.
- Note:Sinceweareapplyingsameamountofenergy(Q)andobservinginthesametime (Δt),numberofatomsexcitedintohigherenergylevels(absorption)=numberofatoms that made transition into lower energy levels (stimulated emission)
  $B_{12}=B_{21}$i.e. absorption=stimulated emission

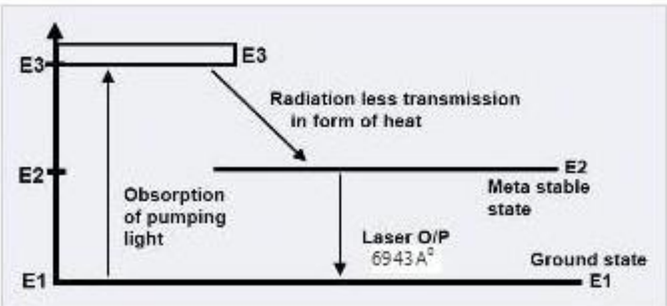
6) Describe the principle, construction and working of ruby laser with relevant energy level diagram?

- **RubyLaser:**Itis a3 levelsolid state laser,discovered byDr.T.Maiman in 1960.
  <u>Principle:</u>
- Thechromium Ionsraisedtoexcitedstatesbyopticalpumpingusingxenonflashlamp
- Thenthe atoms areaccumulated at metastablestate bynon-radiative transition.

- Due to stimulated emission the transition of atoms take place from metastable state to ground state, there by emitting laser beam.
- **Construction:**



- Rubyisacrystalofaluminumoxide(Al_2O_3)inwhichsomeofthealuminumions(Al^{3+}) is replaced bychromium ions (Cr^{3+}). This is done bydoping small amount (0.05%) of chromium oxide (Cr_2O_3) in the melt of purified Al_2O_3.
- Thesechromiumionsgivethepinkcolortothecrystal.Laserrodsarepreparedfroma singlecrystalofpinkruby.Al_2O_3doesnotparticipateinthelaseraction.Itonlyactsas the host.
- Therubycrystalisintheformofcylinder.Lengthofrubycrystalisusually2cmto30 cm and diameter 0.5 cm to 2 cm.
- Theendsof rubycrystalarepolished,grounded andmade flat.
- The one of the ends is completely silvered while the other one is partially silvered to get the efficient output. Thus the two polished ends act as optical resonator system.
- Ahelical flashlampfilledwithxenonisusedasapumpingsource.Therubycrystalis placedinside axenonflashlamp.Thus,opticalpumpingisusedtoachieve population inversion in ruby laser.
- As very high temperature is produced during the operation of the laser, the rod is surrounded by liquid nitrogen to cool the apparatus.
- **WorkingwithEnergyLevelDiagram(ELD):**

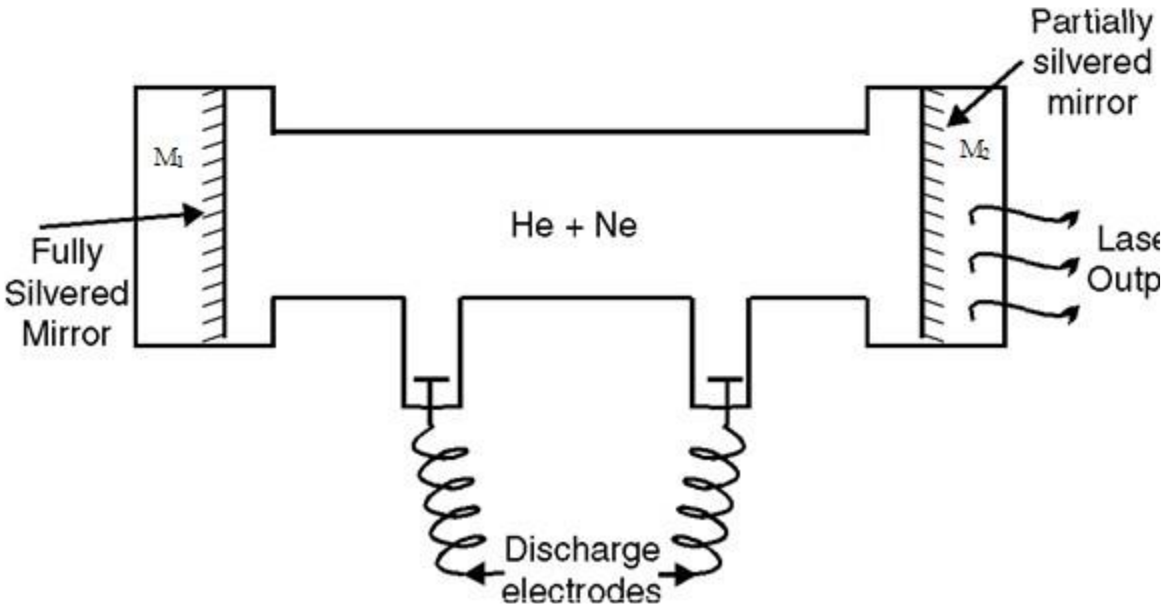


- Fig:EnergyLevelDiagram ofRubyLaser

- The flash lamp is switched on, a few thousand joules of energy is discharged in a few milliseconds.
- A part of this energy excites the Cr^{3+} Ions to excited state from their ground state and the rest heats up the apparatus can be cooled by the cooling arrangement by passing liquid nitrogen.
- The chromium ions respond to this flash light having wavelength 5600 A^0(Green),[4200 A^0(Red)Also]
- WhentheCr^{3+}IonsareexcitedtoenergylevelE$_3$fromE$_1$thepopulationinE$_3$increases.
- Cr^{3+} Ions stay here(E_3) for a very short time of the order of 10^{-8} sec, then they drop to the level E_2which is metastable state of lifetime 10^{-3} sec .Here the transitions from $E_3$to $E_2$is non radiative in nature.
- As the lifetime of the state $E_2$is much longer, the number of ions in this state goes on increasingwhileinthegroundstate(E_1)goesondecreasing.Bythisprocesspopulation inversion is achieved between E_2& E_1.
- When an excited ion passes spontaneously from the metastable state $E_2$to the ground state E_1it emits a photon of wavelength 6943A^0.
- This photon travels through the ruby rod and if it is moving parallel to the axis of the crystal,isreflectedback&forthbysilveredendsuntilitstimulatesanexcitedioninE$_2$and causes it to emit fresh photon in phase with the earlier photon. This stimulated transition triggers the laser Transition.
- The process is repeated again and again, because the photons repeatedly move along the crystal being reflected from ends. The photons thus get multiplied.
- When the photon beam becomes sufficiently intense, such that a part of it emerges through the partially silvered end of the crystal.

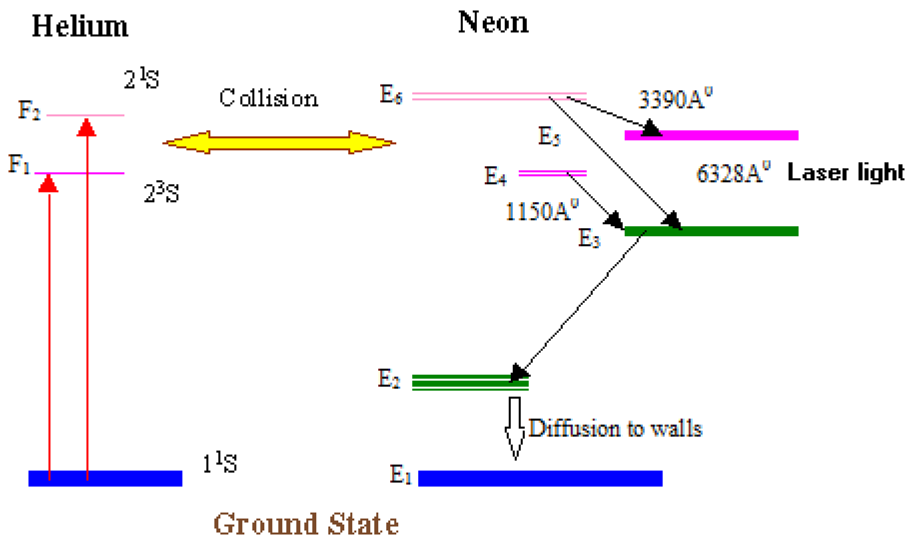
7) Describe the principle, construction and working of He-Ne laser with relevant energy level diagram?

- **He-LeLaser:**
- **Principle:**Thislaserisbasedontheprincipleofstimulatedemission,producedinthe active medium of gas. Here, the population inversion achieved due to the interaction between the two gases which have closer higher energy levels.
- **Construction:**



- Fig:He-Ne laser

- The first gas laser to be operated successfully was the He-Ne laser in 1961 by the scientist A. Jawan.
- In this method, two gases helium & Neon were mixed in the ratio 10:1 in a discharge tube made of quartz crystal.
- The dimensions of the discharge tube are nearly 80 cm length and 1.5 cm diameter, with its windows slanted at Brewster's angle i.e., $\theta = Tan^{-1}(n)$,Where n = refractive index of the window substance.
- ThepurposeofplacingBrewsterwindowsoneithersideofthedischargetubeisto get plane polarized laser output.
- Two concave mirrors M_1& $M_2$are made of dielectric material arranged on both sides of the discharge tube so that their foci lines within the interior of discharge tube.
- OneofthetwoconcavemirrorsM_1isthicksothatalltheincidentphotonsarereflected back into lasing medium.
- The thin mirror M_2allows part of the incident radiation to be transmitted to get laser output.
- <u>Working:</u>



- Fig:(E.L.D)EnergyLevel Diagramcorresponding toHe-Ne laser

- ThedischargetubeisfilledwithHeliumatapressureof1mmofHg&Neonat0.1mm of Hg.
- When electric discharge is set-up in the tube, the electrons present in the electric field make collisions with the ground state He atoms.
- Hence groundstateHe atomsget excitedtothehigherenergylevels$F_1(2S_1)$,$F_2(2S_3)$.
- HereNeatoms areactivecenters.
- TheexcitedHeatomsmakecollisionwiththegroundstateNeatomsandbringtheNe atoms into the excited states E_4& E_6.
- TheenergylevelsE_4&E_6ofNearethemetastablestatesandtheNeatomsaredirectly pumped into these energy levels.
- SincetheNeatoms areexcited directlyinto thelevels E_4&E_6,theseenergylevels are more populated than the lower energy levels E_3&E_5.
- Therefore,thepopulationinversionisachievedbetweenE_6&E_5,E_6&E_3,E_4&E_3
- Thetransitionbetweentheselevelsproduceswavelengthsof3390A^0,6328A^0,1150A^0respec tively.
- Now The Ne atoms undergo transition from $E_3$to $E_2$and $E_5$to E_2in the form of fast decaygivingphotonsbyspontaneousemission.Thesephotonsareabsorbedbyoptical elements placed inside the laser system.
- The Ne atoms are returned to the ground state(E_1) from $E_2$by non radiative diffusion and collision process, therefore there is no emission of radiation.
- Some optical elements placed inside the laser system are used to absorb the IR laser wavelengths 3390 A^0, 1150 A^0.
- Hencetheoutput ofHe-Nelasercontainsonlyasinglewavelengthof 6328A^0.
- ThereleasedphotonsaretransmittedthroughtheconcavemirrorM_2therebyproducing laser.
- Acontinuouslaserbeamofredcolor atawavelengthof6328A^0.
- By the application of large potential difference, Ne atoms are pumped into higher energy levels continuously.
- ALaserbeamofpower0.5to50MWcomesoutfromHe-Nelaser.

8) Describetheprinciple,constructionandworkingofSemiconductorlaserwithrelevant energy level diagram?

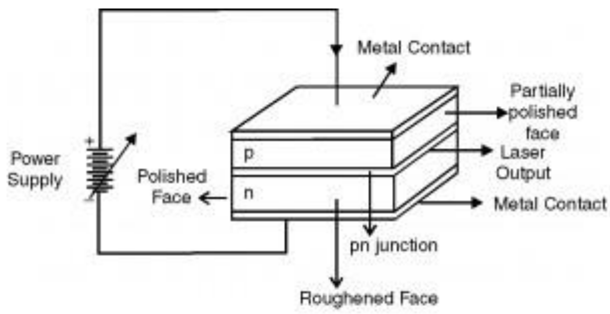
<u>SemiConductorLaser:</u>

- Semiconductorlasersareoftwotypes,ExcepttheConstruction,Principleandworking are same for both.
  1. Homojunctionsemiconductor Laser
  2. HetrojunctionsemiconductorLaser

<u>Principle:</u>

- After the invention of semi conductor leaser in 1961, laser have become at common use.
- Inconventionallasers,lasersaregeneratedduetotransitionofelectronsfromhigherto lower energy level.
- Butinsemi-conductorlasersthetransitiontakesplacefromconductionbandtovalence band.
- Thebasicmechanismresponsibleforlightemissionfromasemiconductorlaseristhe recombinationofe'sandholesatPN-junctionwhencurrentispassedthroughthediode.
- Stimulatedemissioncanoccurwhentheincidentradiationstimulatesaneinconduction band to make a transition into valence band in that process radiation will be emitted.
- When current is passed through PN – junction under forward bias, the injected e's & holes will increase the densityof e in CB &holes in VB. At some value of current the stimulated emission rate will exceed the absorption rate.
- Asthecurrentisfurtherincreasedatsomethresholdvalueofcurrenttheamplification will takes place and laser begin to emit coherent radiation.
- Theproperties ofsemi conductor laserdepends uponthe energygap

- **<u>Fabrication/construction:</u>**



- Fig:HomojunctionSemiconductorLaser

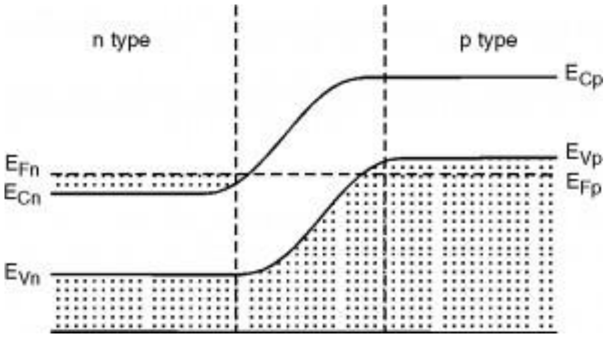
<u>HomojunctionSemiconductorLaser:</u>

- Ga–AsisheavilydopedwithimpuritiesinbothP&Nregions.Nregionisdopedwith tellurium & P – region by Germanium.
- Theconcentration ofdopingis of the order of$10^{17}$to 10^{19}impureatoms per cm.
- Thesizeofthediodeissmalli.e.,1mmeachside&thedepletionlayer'sthickness varies from 1 to 100 μAm.
- Thesevaluesdepend ondiffusion conditionand 40mp atthetimeof fabrication.

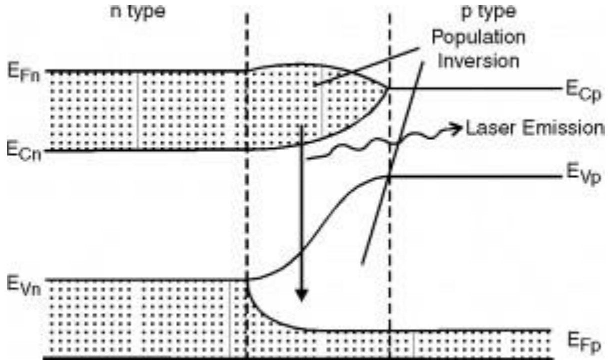
<u>HetrojunctionSemiconductorLaser:</u>

- Hetrojunction means the material on one side of the junction differs from that on the other side.i.e;Ga-AS on one side and GaAlAs on other side.
- Generationamdrecombinationtalesplaceveryfastly.

Working:



Figa)whenno biasing



Figb)with biasing

- Whenaforwardbiaswiththesourceisappliedtoasemiconductor,efromN-region& holes from P-region move to cross the junction in opposite directions.
- Innaturalregionthee's&holescombinerecombinationispossibleduetotransitionof e from CB to VB.
- For low currents the population inversion does not take place hence onlyspontaneous emission takes place and photon released are not coherent.
- Whenforwardcurrentisfurtherincreasedbeyondthecertainthresholdvalue population inversion takes place and coherent photons are released.
- TheenergygapofGalliumArsenide(Ga-As)is1.487eVandcorresponding wavelength of radiation is 6435 A^0 which is responsible for laser emission.

9) MentionsomeimportantapplicationsofLasersinvariousfields?

ApplicationsofLasers: Lasershavewideapplicationsindifferentbranchesofscience and engineering because of the following.

- Verynarrow bandwidth
- Highdirectionality
- Extremebrightness

1. Communication:

- Lasersareusedin opticalcommunications,duetonarrowband width.
- Thelaserbeamcanbeusedforthecommunicationb/wearth&moon(or)other satellites due to the narrow angular speed.
- Usedtoestablishcommunicationbetweensubmarinesi.e;underwatercommunication.

2. Medical:

- Identificationoftumorsandcurification.
- Usedtodetect and removestonesin kidneys.
- Usedtodetecttumorsinbrain.

3. Industry:

- Usedto makeholes indiamond andhardsteel.
- Usedtodetectflawsonthesurfaceof aeroplanesand submarines.

4. Chemical&Biological:

- Lasershavewidechemicalapplications.Theycaninitiateorfastenchemicalreactions.
- Usedintheseparationofisotopes.
- Laserscan beused to findthesize&shapebiological cellssuch aserythrocytes.

10. withthehelpofasuitablediagramexplaintheprinciple,structureandworkingof an optical fiber as a wave guide?

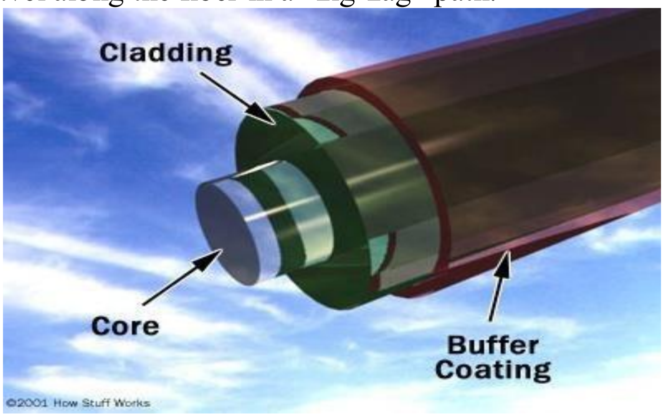
Principle: Optical fibers are the waveguides through which electromagnetic waves of optical frequency range can be guided through them to travel long distances.

- Anopticalfiberworksontheprincipleoftotalinternalreflection(TIR).

Total Internal Reflection: when a ray of light travels from a denser medium into a rarer medium and if the angle of incidence is greater than the critical angle then the light gets totally reflected into the denser medium

Structure&Working:

- Anoptical fiber consists of three(3) co-axial regions.
- Theinnermostregionisthelight-guidingregionknownas"Core".Itissurroundedby a middle co-axial regional known as "cladding". The outer most regions which completely covers the core & cladding regions is called "sheath or buffer jacket".
- Sheath protects the core & cladding regions from external contaminations, in addition to providing mechanical strength to the fiber.
- Therefractiveindexofcore(n_1)isalwaysgreaterthantherefractiveindexofcladding (n_2) i.e., $n_1 > n_2$to observe the light propagation structure of optical fiber.
- Whenlightentersthroughoneendofopticalfiberitundergoessuccessivetotalinternal reflections and travel along the fiber in a "zig-zag" path.



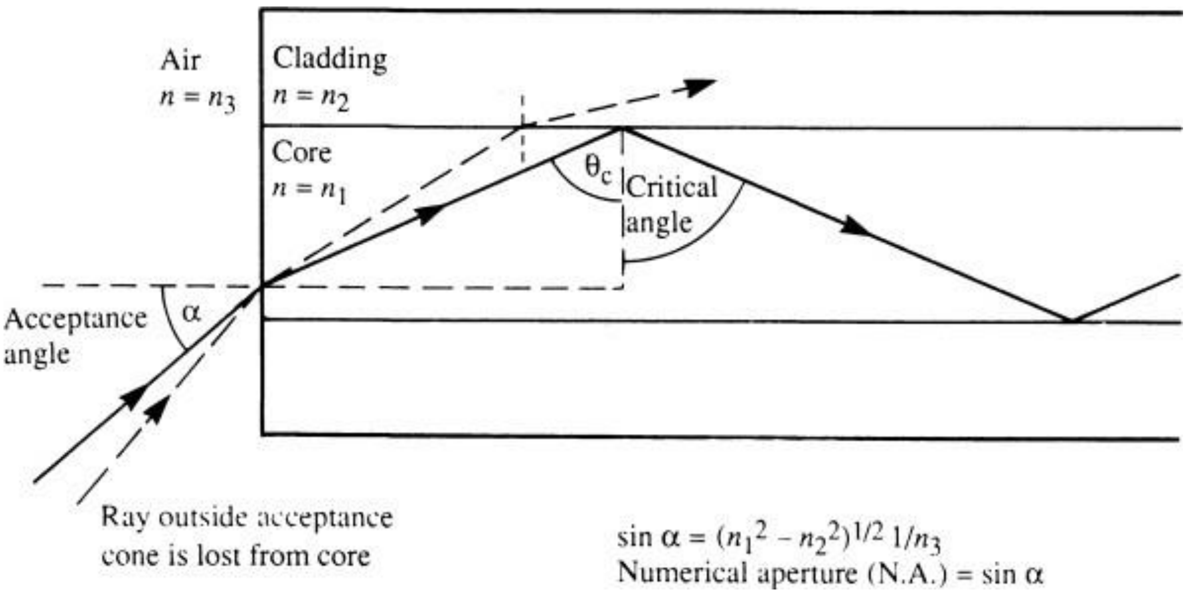
11) Defineandderivetheexpressionsforacceptanceangleand numericalAperture?

Expressionsforacceptanceangle&NumericalAperture(NA):Accep

tance Angle:

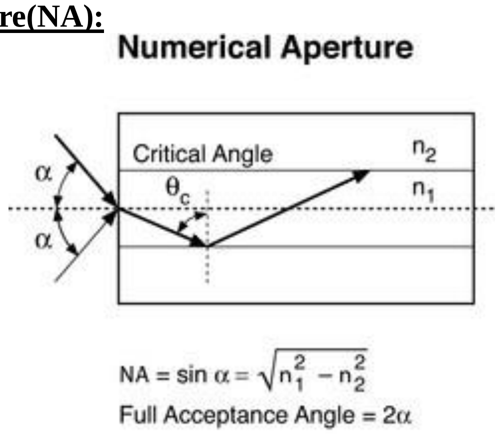
- Allrightraysfallingonopticalfiberarenottransmittedthroughthefiber.Onlythose light rays making $\theta_i > \theta c$ at the core-cladding interface are transmitted through the fiberbyundergoingTIR.Forwhichangleofincidence,therefractionangleisgreater than 90^0 will be propagated through TIR.

- Thereby Acceptance Angle is defined as: The maximum angle of incidence to the axis of optical fiber at which the light ray may enter the fiber so that it can be propagated through TIR.



- Consider the optical fiber with core refractive index n_1 and cladding refractive index n_2. Light is incident on the axis of optical fiber at an angle θ_1. It can produce an angle of refraction θ_2.
- The relationship at the interface is given by snell's law as:
  At air-core interface(A), $n_o \sin\theta_1 = n_1 \sin\theta_2$ -- 1
  At core-clad interface(B), for TIR, $n_1 \sin(90-\theta_2) = n_2 \sin 90^0$
  $n_1 \cos\theta_2 = n_2$, $\cos\theta_2 = n_2/n_1$ -- 2
- Eq'n1 can be written as, $n_0 \sin\theta_1 = n_1\sqrt{1-\cos^2\theta_2}$ -- 3
- Substituting 3 in 2, $n_0 \sin\theta_1 = n_1\sqrt{1-n_2^2/n_1^2}$
  $n_0 \sin\theta_1 = \sqrt{n_1^2 - n_2^2}$
- For air $n_0=1$, then $\sin\theta_1 = \sqrt{n_1^2-n_2^2}$
- $\theta_1 = \theta_A = \sin^{-1}\sqrt{n_1^2-n_2^2}$, Here θ_A is called Acceptance angle
- This gives max value of external incident angle for which light will propagate in the fiber.

NumericalAperture(NA):



- Numericalapertureof afiberis ameasureof itslightgatheringpower.
- “TheNumericalAperture(NA)isdefinedasthesineofthemaximumacceptance angle”
- Thelightgatheringabilityofopticalfiberdependsontwofactorsi.e., Core diameter & NA.
- NAis defined assineof theacceptanceanglei.e., $NA=\sin\theta_A$
  $NA=\sqrt{n_1^2-n_2^2}$
- The efficiency of optical fiber is expressed in terms of NA, so it is called as figure of merit of optical fiber.
- # NAis also expressed like this: $NA = \sqrt{n_1^2-n_2^2} = \sqrt{(n_1-n_2)(n_1+n_2)}$
- Fractionalindexchange$\Delta=n_1-n_2/n_1=n_1-n_2=\Delta n_1$
  Then $NA = \sqrt{\Delta n_1(n_1+n_2)}$
  Let$n_1=n_2$,then$n_1+n_2=2n_1$
- Then$NA=\sqrt{\Delta n_1-2n_1}=n_1\sqrt{2\Delta}=n_1\sqrt{2\Delta}$

12) Howopticalfibersareclassifiedon thebasisofrefractiveindex profile?

Or

DescribetheStepindexandgradedindexopticalfibersindetailandexplainthetransmission of signal through them?

ClassificationofOptical Fibers:

- Based onvariationin thecorerefractiveindex (n_1),optical fibersaredividedintotwo types
  1. Stepindexfiber
  2. Gradedindexfiber
- Basedonmodeofpropagation,fibersarefurther classifiedin to
  1. Singlemode propagation
  2. Multimode propagation
- Stepindexfibershavebothsingle&multimode propagations.
- Gradedindexfibershavemultimodepropagation only
- Alltogetherintotal three (3)typesof fibers
  1. Singlemode stepindexfiber
  2. Multimodestep indexfiber
  3. Multimodegradedindex fiber

Transmissionof Signalin OpticalFibers:

1. StepIndexFiber: Therefractiveindexofcorematerialisuniformthroughoutand undergoes a sudden change in the form of step at the core-clad interface.

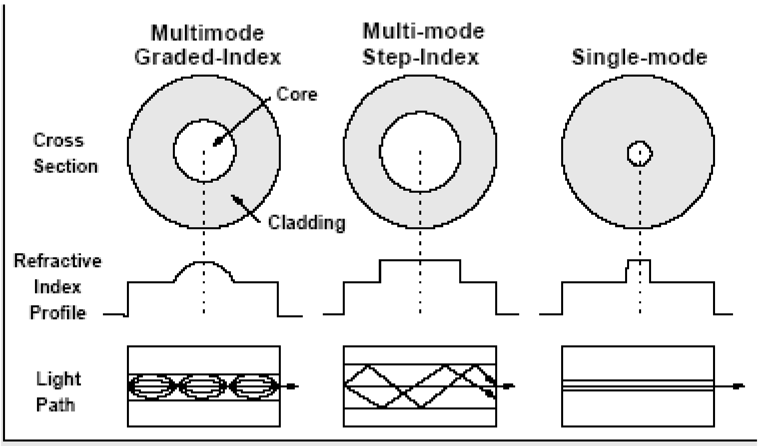


Fig:Refractiveindexprofile&propagationinsinglemode,stepindex&gradedindexfibers

a) SingleModeStepIndexFiber:

- The variation of the refractive index of a step index fiber as a function of distance can be mathematically represented as longitudinal cross section.

Note: Mode of propagation: It is defined as the number of paths available for the light rayto transfer through the optical fiber.

Structure:

i) CoreDiameter: 8 to12µm, usually8.5µm

ii) CladdingDiameter: Around 125µm

iii) SheathDiameter:250to1000µm

iv) NA: 0.08 to 0.15 usually 0.10

PerformanceCharacteristics:

i) BandWidth: Greaterthan500 MHZKm.

ii) Attenuation:2to5 dB/ Km.

iii) Applications: These fibers are ideallysuited for high band width applications using single mode injection coherent (LASER) sources.

B) MultiModeStepIndex Fibers:

- These fibers have reasonably large core diameters and large NA to facilitate efficient transmission to incoherent or coherent light sources.
- Thesefibersallowfinitenumberofmodes.
- Normalized frequency (NF) is the cut off frequency, below which a particular modecannot exist. This is related to NA, Radius of the core, and wave length of light as
  $NF=2\pi/\lambda a(NA)$,Wherea=radius of core

Structure:

i) CoreDiameter: 50 to 200µm

ii) CladdingDiameter: 125 to 400µm

iii) SheathDiameter:250to1000µm

iv) NA:0.16 to 0.5

PerformanceCharacteristics:

i) BandWidth: 6 to50 MHZKm.

ii) Attenuation:2.6to50db/km.

iii) Applications:Thesefibersareideallysuitedforlimitedbandwidthandrelativelylowcost applications.

c) MultiModeGradedIndexFibers:

- Incaseofgradedindexfibers,therefractiveindexofcoreismadetovaryasafunction of radial distance from the centre of the optical fiber.

- Refractive index increases from one end of core diameter to center and attains maximum value at the centre. Again refractive index decreases as moving away from center to towards the other end of the core diameter.
- Therefractiveindexvariationisrepresentedasn(r)=$n_1(1-2\Delta)^{1/2}=n_2$Here
  Δ = fractional change in refractive index = n_1-n_2/n_1
- ThenumberofmodesisgivenbytheexpressionN=4.9[d(NA)/ $\lambda]^2$
  Where d = core diameter, λ = wavelength of radiation

Structure:

i) CoreDiameter: 30 to 100µm

ii) CladdingDiameter: 105 to 150µm

iii) SheathDiameter:250to1000µm

iv) NA:0.2to 0.3

PerformanceCharacteristics:

i) BandWidth: 300 MHZKmto 3 GHZKm.

ii) Attenuation:2to10dB/km.

iii) Applications: These are ideally suited for medium to high band width applications using incoherent and coherent multimode sources.

13) DistinguishStepindex&GradedindexfibersAndSinglemode&Multimodefibers?

Step Index	**GradedIndex**
1. RI of core is uniform throughout except at one stage. 2. Single&multimodepropagationsexist. 3. Usedforshortdistanceapplications. 4. Attenuationlossesareoftheorder100 dB/km. 5. Mer4dinolrayspropagationtakesplace. 6. Easyto manufacture.	1. Refractiveindexvariesgraduallywith radial distance. 2. Itisamultimodefiber. 3. Used forlongdistanceapplications. 4. 4. Attenuation losses are of the order 10 dB/km. 5. Skewrayspropagation takes place. 6. Difficulttomanufacture.
SingleMode	**MultiMode**
1. Corediameteris small. 2. Signal entryis difficult. 3. Existsin step indexfiber. 4. Lightmustbe coherent.	1. Corediameteris large. 2. Signalentryis easy. 3. Existsinbothstep&gradedindexfibers. 4. Light source may be coherent or incoherent source .

14) Whatarethe Advantagesof opticalfibersover metalliccables?

- Optical fibers allow light signals of frequencies over a wide range and hence greater volumeofinformationcanbetransmittedeitherindigitalformorinanalogformwithin a short time.
- Inmetalliccablesonly48conversationscanbemadeatoncewithoutcrosstalkswhere as in optical fibers more than 15000 conversations can be made at once without cross talks.
- Light cannot enter through the surface of the optical fiber except at the entryinterface i.e.,interferenceb/wdifferentcommunicationchannelsisabsent.Hencepurityoflight signal is protected.

- Optical signal do not produce sparks like electrical signals and hence it is safe to use optical fibers.
- ExternaldisturbancesfromTVorRadioStationspowerelectronicsystemsandlightening cannot damage the signals as in case of metallic cables.
- Materials used in the manufacture of optical fibers areSiO_2, plastic, glasses which are cheaper & available in plenty.

15) Howopticalfibersareusedincommunicationfield?OrExplainopticalfibercommunication link with help of block diagram.

OpticalFiberCommunicationLink:

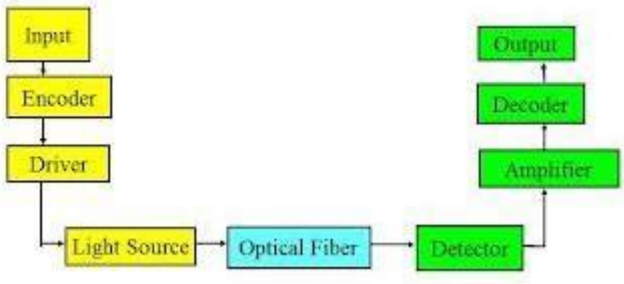


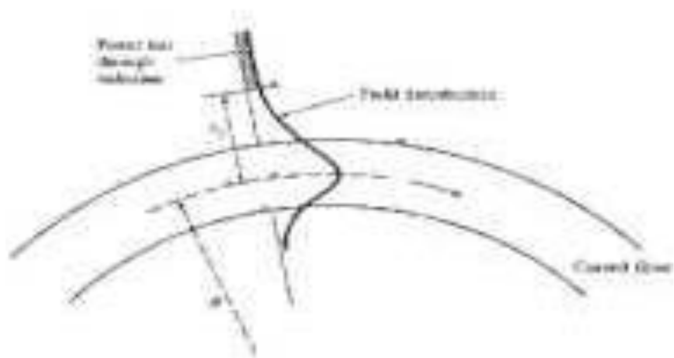
Fig:BlockDiagramofOpticalfibercommunicationlink

Optical fiber is an ideal communication medium by systems that require high data capacity, fast operation and to travel long distances with a minimum number of repeaters.
Encoder: Itisanelectronicsystemthatconvertstheanaloginformationsignals,suchasvoice of telephone user, in to binary data. The binary data consists of series of electrical pulses.
Transmitter: Transmitter consists of a driver which is a powerful amplifier along with light source. The o/p of amplifier feeds to light source, which converts electrical pulses in to light pulses.
SourcetoFiberConnector: Itisaspecialconnectorthatsendsthelightfromsourcestofiber. Theconnectoractsastemporaryjointb/wthefiberandlightsource,misalignmentofthisjoint, leads to loss of signal.,
FibertoDetectorConnector: Itisalsotemporaryjoint,whichcollectsthesourcefromfiber.
Receiver: Receiver consists of a detector followed by amplifier. This combination converts light pulses in to electrical pulses.
Decoder: Electrical pulses containing information are fed to the electronic circuit called decoder. Decoder converts binary data of electrical pulses in to analog information signals.

16) Writeashort noteonattenuation in optical fibers.

Usually,thepowerofflight attheoutputendofopticalfiberisless thanthepowerlaunched at the input end, then the signal is said to be attenuated.
Attenuation: Itistheratioofinputopticalpower(P_i)intothefibertothepoweroflight coming out at the output end (P_o).
Attenuationcoefficientisgivenas,α=10/L$\log_{10}P_i/P_o$db/km.
Attenuation is mainly due to
1. Absorption.
2. Scattering.
3. Bending.

1. <u>AbsorptionLosses</u>: Inglassfibers,threedifferentabsorptionstake place.
Ultra violet absorption: Absorption of UV radiation around 0.14μm results in the ionization of valence electrons.
Infrared absorption: Absorption of IR photons by atoms within the glass molecules causes heating. This produces absorption peak at 8μm, also minor peaks at 3.2, 3.8 and 4.4μm.
Ionresonance/OH^-absorption: The OH^-ions of water ,trapped during manufacturing causes absorption at 0.95, 1.25 and 1.39μm.
2. <u>ScatteringLosses</u>:
The molten glass,when It isconvertedintothinfiberunderpropertensioncreatessub microscopic variations in the density of glass leads to losses.
Thedopents addedtotheglasstovarytherefractiveindex alsoleadstotheinhomogenitiesin the fiber. As a result losses occur.
Scatteringlossesareinverselyproportional tofourth powerofλ.(λ^4)
3. <u>BendingLosses:</u>
In a bent fiber, there is a loss in power of the transmitted signal called as Bending Loss. According to the theory of light, the part of the wavefront travellingincladding(rarermedium) should travel with more velocity than the wave front in the core (denser medium). But according to Einstein's theory of relativity, in a single wave front one part should not travel with higher velocity than the other part.
So the part of wave front travelling in cladding medium lost in the form of radiation leads to bending losses.



Faraday's law of electromagnetic induction:

Faraday stated two laws from the observations of Oersted study is called Faraday's law of electromagnetic induction

1) **Whenever the magnetic flux linked with an electric circuit (coil) changes, an e.m.f is induced in the circuit (coil). The induced e.m.f exists as long as the change in magnetic flux continues.**

2) **The magnitude of induced e.m.f is directly proportional to the negative rate of variation of the magnetic flux linked with the circuit.**

If Φ_B be the magnetic flux linked with circuit at any instant and e be the induced e.m.f then

$$e = -\frac{d\Phi_B}{dt} \quad \rightarrow (1)$$

Your text here 1

If there are N turns in the coil, then

$$e = -N\frac{d\Phi_B}{dt} \quad \rightarrow \quad (2)$$

The negative sign is in accordance with Lenz's law, this is also called **Neumann's Law**

Lenz's Law:

The Lenz's law is based on the principle of conservation of energy. Thus it helps to explain the direction (polarity) of induced e.m.f or induced current in a coil.

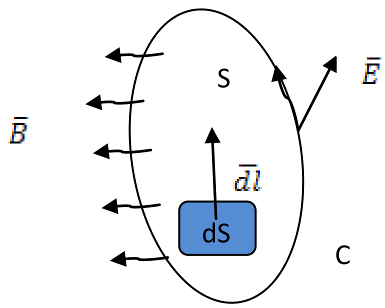
The polarity of the induced e.m.f is always such that it tends to produce a current which opposes the change in magnetic flux that produced it.

The changing magnetic field and magnetic flux induces an electric current in a coil. This induced current itself creates magnetic field and hence magnetic flux is induced around the coil. Therefore the change in the external magnetic field and flux is always opposed.

Vector form of Faraday's law(Integral and Differential forms):

Integral form:

Consider that magnetic field is produced by a stationary magnet or current carrying coil. Suppose there is a closed circuit C of any shape which encloses a surface S in the field as shown in fig., Let $\bar{B}$ be the magnetic flux density in the neighborhood of the circuit.



The magnetic flux through a small area "dS" will be $\bar{B}$. Now the flux through the entire circuit is

$$\Phi_B = \int_S \bar{B}\,.dS \quad \rightarrow(1)$$

When the magnetic flux is changed, an electric field $\bar{E}$ induced around the circuit. The line integral of the electric field gives the induced e.m.f in the closed circuit. Thus

$$e = \oint E.dl \quad \rightarrow(2)$$

Where E is the electric field at an element of dl of the circuit. Substituting the values of e and Φ_B from equations (2) and (1) we have

$$\oint E.dl = -\frac{d\Phi_B}{dt} = = -\frac{d}{dt}\int_S B.ds \quad \rightarrow(3)$$

This is the integral form of Faraday's law.

Differential form:

According to equation (3) the line integral of the electric field around any closed circuit is equal to the negative rate of change of magnetic flux through the circuit.

Further by stokes theorem we have

$$\oint E.dl = \int_S (\nabla\times E).dS \quad \rightarrow(4)$$

From equation (3) and (4) we get

$$\int_S (\nabla\times E).dS \quad = -\frac{d}{dt}\int_S B.ds$$

Hence $\nabla\times E = -\frac{dB}{dt}$

This is the differential form of Faraday's law.

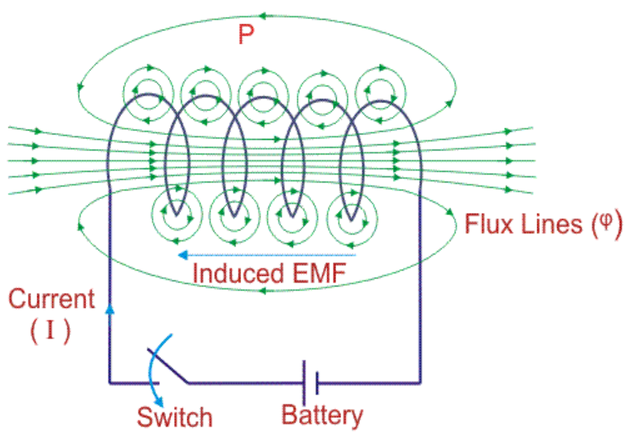
Self Induction:

The phenomenon of self induction was discovered by J. Henry in 1932. When a transient current passes through a coil it produces magnetic field around it, flux due to this field is linked with the coil itself. Due to its own flux change, an e.m.f is induced in the coil and it is called induced e.m.f or back e.m.f.

Definition: Self induction is the property of a coil by virtue of which opposes the growth or decay of the current flowing through it.

(OR)

Self inductance is the phenomenon of inducing e.m.f in a coil due to flow of current which changes with time in the same coil



Consider a coil connected to a battery through key (k). When the key is closed, due to increasing current in the coil, the magnetic field and hence flux linkage with the coil also increases. As a result of this, induced emf is set up in the coil. According to Lenz's law, the direction of induced emf is such that it opposes the growth of current in the coil. This delays the current to acquire the maximum value.

When the key is released, the current in the coil starts decreasing, so the magnetic flux linked with the coil decreases. As a result of this change in the magnetic flux, induced emf is set up in the coil itself. According to Lenz's law, the direction of induced emf is such that it opposes the decay of the current in the coil. This delays the current to acquire minimum (or) zero value.

NOTE: Self induction is also known as inertia of electricity as it opposes the growth or decay of the current in the circuit

Coefficient of self induction (or) self inductance:

The total magnetic flux Φ_B linked with the coil is proportional to the current I flowing in it i.e.,

$$\Phi_B \propto i$$

Or $$\boxed{\Phi_B = Li} \quad \rightarrow (1)$$

Where L is a constant called the coefficient of self induction or self inductance of the coil. When i =1 , Φ_B =L. Hence the coefficient of self induction is numerically equal to the magnetic flux linked with the coil when unit current flows through it.

The e.m.f. induced in the coil is given by

$$e = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d(Li)}{dt}$$

$$= -L\frac{di}{dt} \quad \rightarrow (2)$$

The negative sign indicates that the induced e.m.f is in such a direction as to oppose the change

When $\frac{di}{dt}$ =1 ; e= -L

Therefore the coefficient of self inductance is numerically equal to the induced e.m.f in the coil, when the rate of change of current is unity.

Unit: The unit of self inductance is henry which is inductance of a coil in which an e.m.f of 1 volt is set up by the change of current at 1 ampere per second

$$1 \text{ henry} = \frac{1\ volt}{1\ amp/sec}$$

Energy stored in electric fields:

Consider a capacitor of capacitance C and carrying a charge q at any instant. Let the potential difference between the plates V, then

$$V = q/c \quad \rightarrow (1)$$

If an additional charge dq is to be given to this capacitor then some work must be done against the potential difference.

So the work done increasing charge by dq is given by

$$dW = V\,dq = (q/C)\,dq \quad \rightarrow (2)$$

∴ Total work to charge a capacitor to a charge q_0

$$W = \int dW = \int_0^{q_0} \frac{q}{C}\,dq = \frac{{q_0}^2}{2C} \quad \rightarrow (3)$$

Now the energy stored by a charged capacitor

$$U = W = \frac{1}{2}\frac{q^2}{C} = ½(CV^2) \quad (\because q = CV) \quad \rightarrow (4)$$

For a parallel plate capacitor of area A and plate separation d, the capacitance C is given by

$$C = \frac{\varepsilon_0 A}{d} \quad \text{and } V = Ed$$

∴ Energy stored $U = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times E^2 d^2$

$$\boxed{U = ½\,(\varepsilon_0 E^2 Ad)\ \text{joules}}$$

Energy stored in magnetic fields:

When the current in a coil is switched on, self induction opposes the growth of current i.e. the current flows against back e.m.f and does work against it.

$$dW = -\,e\,i\,dt$$

$$dW = +L\frac{di}{dt}\,i\,dt \quad \left(\because e = -L\frac{di}{dt}\right)$$

Hence, total work done in bringing the current from zero to a steady maximum value i_0 is

$$W = L\int_0^{i_0} i\, di$$

$$W = \frac{1}{2}(Li_0^2)$$

Consider a very long solenoid of length 'l' and cross-sectional area 'A'. When current flows in it, a magnetic field is established and the work done is stored as energy in the magnetic field given by

$$U = \frac{1}{2}(Li_0^2)$$

But inductance in coil is given by $L = \mu_0 n^2 Al$

Where n is number of turns in solenoid per meter.

$$\therefore U = \frac{1}{2}(\mu_0 n^2 Al\, i_0^2) = \frac{1}{2} \times \frac{(\mu_0 n i_0)^2}{\mu_0} \times Al$$

But the magnetic field inside a coil $B = \mu_0 n i_0$

$$U = \frac{B^2}{2\mu_0} \times Al$$

OR

$$U = \frac{\mu_0 H^2}{2} \times Al$$

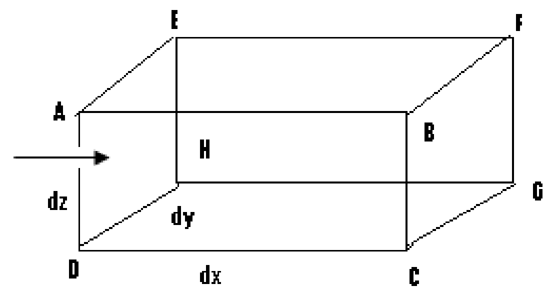
Poynting Vector:

One important characteristic of electromagnetic waves is that they transport energy from one point to another point.

The amount of field energy passing through unit area of the surface perpendicular to the direction of propagation of energy is called as Poynting vector.

The Poynting vector is denoted by $\bar{P}$ given by

$$\bar{P} = \frac{1}{\mu_0}(\bar{E} \times \bar{B}) \quad \text{or} \quad (\bar{E} \times \bar{H})$$



Proof: Consider an elementary volume in the form of a rectangular parallopiped of sides dx, dy and dz as shown in fig. The volume of parallopiped is “dxdydz”. Suppose the energy is propagated along X-axis , now the area perpendicular to the direction of propagation of energy is dydz. Let the electromagnetic energy in this volume is U. Then the rate of change of energy is $\frac{\partial U}{\partial t}$.

$$\frac{\partial U}{\partial t} = -\oint_S P.ds \quad \rightarrow (1)$$

Negative sign is used to show that energy is entering in the volume.

Since the energy per unit volume in electric magnetic fields

$$U = (\tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2) \quad \rightarrow (2)$$

The rate of decrease of energy in volume dV is given b

$$-\frac{\partial U}{\partial t} = -\frac{\partial}{\partial t}\int_V (\tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2)\, dV \quad \rightarrow (3)$$

$$= \int_V -\left[\varepsilon_0 E(\frac{\partial E}{\partial t}) + \mu_0 H(\frac{\partial H}{\partial t})\right] dV \quad \rightarrow (4)$$

From Maxwell’s equation

$$\frac{\partial E}{\partial t} = \frac{\nabla \times H}{\epsilon_0} \text{ and } \frac{\partial H}{\partial t} = -\frac{\nabla \times E}{\mu_0} \quad \rightarrow (5)$$

From equation (5) and (4)

$$-\frac{\partial U}{\partial t} = \int_V [\, H.(\nabla \times E) - E.(\nabla \times H)]\, dV \quad \rightarrow (6)$$

$$= \int_V \nabla.(E \times H) dV \quad \rightarrow (7)$$

From Gauss theorem of divergence (7) can be written as

$$= \oint_S (E \times H) \bullet \hat{n} ds \quad \rightarrow (8)$$

Comparing (8) and (1)

$$\bar{P} = \bar{E} \times \bar{H}$$

The vector shows that energy flow takes place in a direction perpendicular to the plane containing E and H

Characteristics of Poynting Vector

(1) The poynting vector is perpendicular to both electric field vector E and magnetic field vector H
(2) The quantity∇**.** P represents the net energy flow in electromagnetic field
(3) In alternating fields

$$\boldsymbol{P}_{avg} = \boldsymbol{E}_{rms} \times \boldsymbol{H}_{rms}$$

Fundamental laws of electromagnetism:

(1) Gauss law in electrostatics:

The electric flux Φ_E through a closed surface is equal to (1/ε) times the net charge q enclosed by the surface. i.e. The surface integral of the normal component of the electric field $\bar{E}$ over any closed surface equals (1/ε times) the net charge with that volume

$$\Phi_E = \oint \bar{E}\ ds = q/\varepsilon$$

(2) Gauss law in magnetostatics:

The magnetic lines of force leaving from the closed surface will be equal to the number of lines of force entering the surface. i.e. The net outward magnetic flux from any closed surface is zero.

$$\Phi_B = \oint \bar{B}\ ds = 0$$

(3) Faraday's law of electromagnetism

The magnitude of induced e.m.f is directly proportional to the negative rate of variation of the magnetic flux linked with the circuit.

$$\text{e.m.f. (e)} = -\frac{d\phi_B}{dt} \quad \text{(OR)} \quad \oint \bar{E}\cdot dl = -\int_s \frac{\partial B}{\partial t}\bullet ds$$

(4) Ampere's circuit law(or) Biot-Savart's law:

The steady current carrying conductor generates magnetic field around it

$$\oint \bar{B}\cdot dl = \mu_0 i$$

Basic equations of electromagnetism- Maxwell's equation

Maxwell's Law	Integral form	Differential form
First law (based on Gauss law of electrostatics)	$\oint \nabla \cdot \bar{D}\ ds = \int_V \rho \bullet dV$	$\nabla \cdot \bar{D} = \rho$
Second law (based on Gauss law of magnetostatics)	$\oint \bar{B}\ ds = 0$	$\nabla \cdot \bar{B} = 0$
Third law (based on Faraday's law of electromagnetic induction)	$\oint \bar{E} \cdot dl = -\int_S \frac{\partial B}{\partial t} \bullet ds$	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$
Fourth law (based on Ampere's circuital law)	$\oint H \cdot dl = \int_S (J + \frac{\partial D}{\partial t}) \bullet ds$	$\nabla \times \bar{B} = \mu_0 (j + \frac{\partial \bar{D}}{\partial t})$

Displacement current:

According to Maxwell Fourth law, steady current carrying conductor generates magnetic field around it.

$$\oint \bar{B}.\bar{dl} = \mu_0 i \qquad \rightarrow (1)$$

Hence

$$\oint B.dl = \mu_0 \int_S j \bullet ds \qquad \rightarrow (2)$$

Using Stoke's theorem $\oint B.dl = \int_S \nabla \times \bar{B} \bullet dS \quad \rightarrow (3)$

From (2) and (3) $\int_S \nabla \times \overline{B} \bullet dS = \mu_0 \int_S j \bullet ds$ →(4)

Or $\nabla \times \bar{B} = \mu_0 j$ → (5)

Taking divergence of this equation we get

$\nabla.(\nabla \times B) = \text{div}(\text{curl } B) = \text{div}\mu_0 j = \mu_0 j$ →(6)

However that divergence of curl of a vector is always zero and hence

div j = 0 →(7)

This shows that the total flux of current out of any closed surface is zero. However from equation of continuity

$\text{div } j + \frac{\partial \rho}{\partial t} = 0$ →(8)

The equation (8) contradicts equation(7)

From Maxwell I equation $\nabla.\, D = \rho$

Also $\text{div } j + \frac{\partial \rho}{\partial t} = \text{div}\left(j + \frac{\partial D}{\partial t}\right)$

Thus $\nabla.j = 0$ for steady current

And $\nabla.\left(j + \frac{\partial D}{\partial t}\right) = 0$ every where

The term $\frac{\partial D}{\partial t}$ is called as displacement current density.

Thus a changing electric field is equivalent to a current which flows as long as the electric field is changing and produced the same magnetic effect as an ordinary conduction current. This is known as displacement current.

Electromagnetic wave equation:-

According to Maxwell's electromagnetic equations in a homogeneous medium

(i) It has infinite resistance to the current and hence its conductivity is zero i.e. **j** = 0
(ii) It has no volume distribution of charge, thus the charge density $\rho = 0$
(iii) Also $\bar{D} = \epsilon \bar{E}$ and $\bar{B} = \mu \bar{H}$

Hence Maxwell equations

$\nabla \cdot \bar{E} = 0$ →(1)

$$\nabla \cdot \bar{B} = 0 \quad \rightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \rightarrow (3)$$

$$\nabla \times \bar{B} = \mu\varepsilon \frac{\partial \bar{E}}{\partial t} \quad \rightarrow (4)$$

The wave equation of propagation of a wave can be obtained by taking curl of eq.(4) as

$$\nabla \times \nabla \times \bar{B} = \nabla \times \mu\varepsilon \frac{\partial \bar{E}}{\partial t} = \mu\varepsilon \left(\nabla \times \frac{\partial \bar{E}}{\partial t}\right)$$

$$= \mu\varepsilon \frac{\partial}{\partial t}(\nabla \times \bar{E}) \qquad (\because \text{ from eq.(3)})$$

$$= \mu\varepsilon \frac{\partial}{\partial t}\left(-\frac{\partial \bar{B}}{\partial t}\right)$$

$$= -\mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2} \qquad \rightarrow (5)$$

Thus $\nabla \times \nabla \times \bar{B} = \nabla(\nabla . \bar{B}) - \nabla^2 \bar{B} = -\nabla^2 \bar{B}$ $\rightarrow (6)$

From (5) and (6)

$$-\nabla^2 \bar{B} = -\mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2}$$

Or $\nabla^2 \bar{B} = \mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2}$ $\rightarrow (7)$

Similarly, from eq.(3) we can show that

$$\nabla^2 \bar{E} = \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} \qquad \rightarrow (8)$$

From eqs.(7) and (8) represents the relation between the space and tie variation of magnetic field $\bar{B}$and electric field $\bar{E}$. Hence the general wave equation is represented by

$$\boxed{\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \qquad \rightarrow (9)$$

$$\frac{1}{v^2} = \mu\varepsilon \qquad \rightarrow (10)$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Where μ and ε are permeability and permittivity of the medium

When the electromagnetic wave propagating in free space

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\mu_0 = 4\pi\times10^{-7}$ and $\varepsilon_0 = 1/(4\pi\times9\times10^{9})$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^{9}}}}$$

$$v = \sqrt{9 \times 10^{16}} = 3\times10^{8} \text{ m/s}$$

Thus the velocity of propagation of variation of **E** and **B** is the same as the velocity of light.

Equation (9) indicate the wave propagation in 3-D free space. These waves involve periodic variations of electric and magnetic field. So they are called electromagnetic waves.

Ultrasonic's: Ultrasonic sound waves have frequencies above the human ear's audible range that is greater than 20kHz and often into megahertz range.

PRODUCTION OF ULTRASONIC WAVES:

The ultrasonic waves cannot be produced by our usual method of loudspeaker fed with alternating current. This is due to the fact that at very high frequencies. There are two methods namely magnetostriction and piezoelectric are used to produce the ultrasonic's

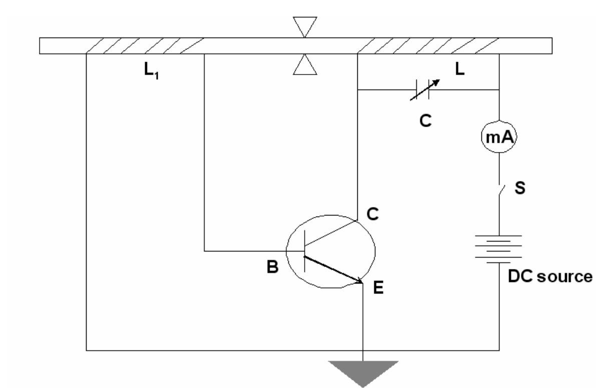
Magnetostriction method

Principle:

When an alternating magnetic field is applied parallel to the length of a ferromagnetic rod such as iron or nickel, a small elongation or contraction occurs in its length. This phenomenon is known as magnetostriction.

Construction:

An experimental arrangement due to production of ultrasonic waves is shown in fig. There is a short nickel rod which is clamped at the centre. A simple tuned oscillator constructed with a NPN transistor with L-C circuit is connected in the collector



Working:

When the supply is switched on, collector current starts rising and oscillations start in the L-C circuit, the changes of current in inductor L is feedback to the base emitter circuit through mutual inductance between L and L1. The transistor merely ensures that energy is feedback at the correct phase from the source. The frequency of oscillation of L-C circuit is given by $f = 1/2\pi\sqrt{(LC)}$. By varying C, the frequency can be adjusted to be in tune with the natural frequency of the rod. Under the resonance condition sustained oscillations and hence ultrasonic waves are produced by the rod.

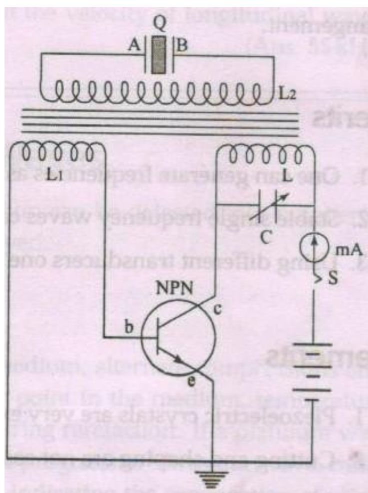
Piezoelectric method:

Principle:

In certain crystals such as quartz is subjected to pressure on one pair of opposite faces (mechanical faces) then in the other pair of opposite faces (electric faces), an opposite electric charges are developed. Similarly when the electric faces are subjected to the alternating voltage then the mechanical faces produces frequencies of order more than 100 kHz

Construction:

The circuit diagram used for generating ultrasonic waves using piezoelectric effect is shown in fig. Q is a thin slice of quartz crystal cut with its opposite faces perpendicular to optic axes. The crystal Q is placed between two metal plates A and B which act as electrodes. The plates are connected to the coil L_2 Coil L is connected to the collector circuit.



Working :

When the supply is switched on, collector current starts raising and oscillations start in the L-C circuit. The frequency of oscillation of the L-C circuit is given by the expression $f = 1/2\pi\sqrt{(LC)}$. By varying C the frequency of oscillation of the circuit can be adjusted such that electrodes A and B connected to the coil L_2 are induced with alternating e.m.f. Hence the crystal

Q placed between the electrodes A and B experience oscillating electric force and due to inverse piezoelectric effect, high frequency ultrasonic waves are produced.

APPLICATIONS:

1) Ultrasonic inspection is used for quality control and material applications
2) Used to measure thickness of metal sections
3) Thickness measurements are made on refinery and chemical procession equipments, submarine hulls, aircraft sections and pressure vessels.
4) Used to detect internal corrosion.