

TOPIC :- Introduction of quantum Mechanics

OBJECTIVES:- To understand the Introduction of quantum mechanics

OUTCOME:- Stu. will be able to introduce quantum mechanics.

PHYSICS

CLASSICAL PHYSICS

MODERN PHYSICS

It deals with macroscopic particle which is visible.

QUANTUM MECHANICS

It deals with microscopic particle which is not visible

It deals with mechanical behaviour of submicroscopic particle.

INTRODUCTION TO QUANTUM MECHANICS.

- Wave theory of light
- Explain Optical phenomenon
- such as

* Interference...

* Diffraction...

* Polarization...

BUT, WAVE THEORY FAILS TO EXPLAIN :-

- ⊗ COMPTON EFFECTS
- ⊗ PHOTOELECTRIC EFFECTS
- ⊗ ABSORPTION OF LIGHT BY THE SUBSTANCE
- ⊗ BLACK BODY RADIATION ETC

- PLANCK QUANTUM THEORY •

- According to this theory, light is in the form of packet of energy

- called quanta or photon

- What is Quanta?

- Packet of Energy

$$E = h\nu$$

$$p = \frac{h\nu}{c}$$

This theory explains to :

- * COMPTON EFFECTS.

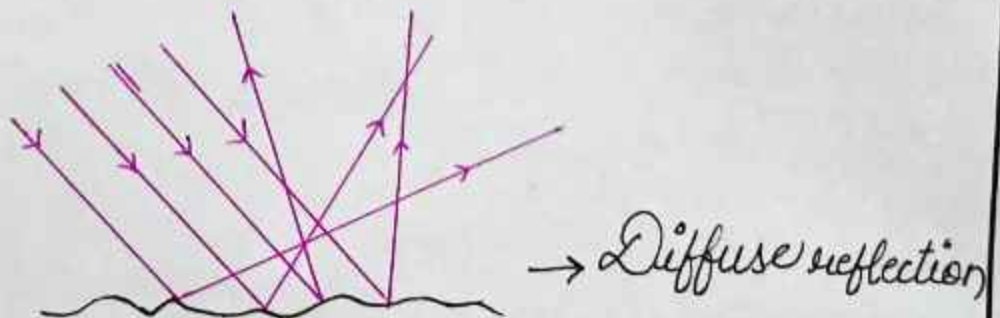
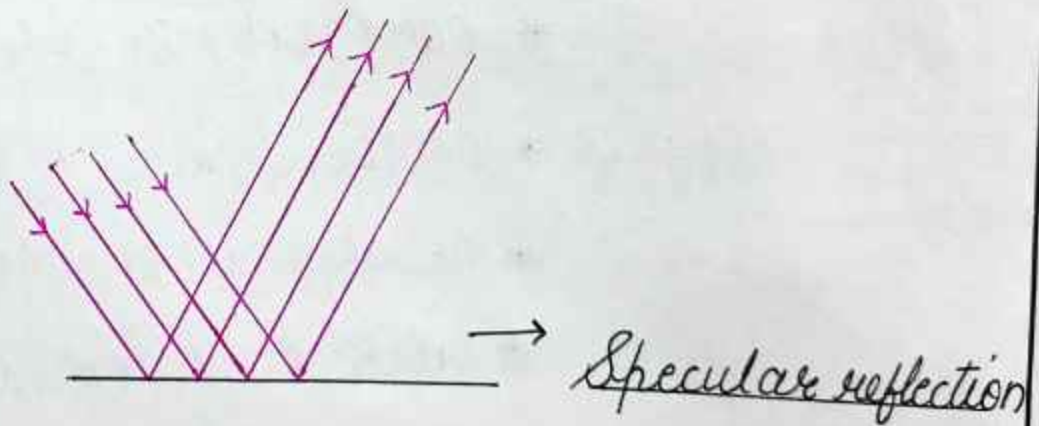
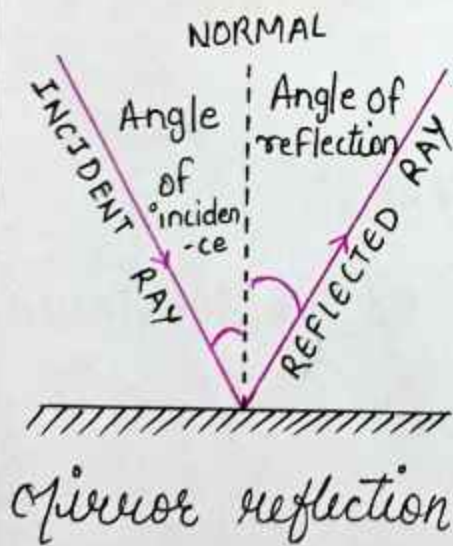
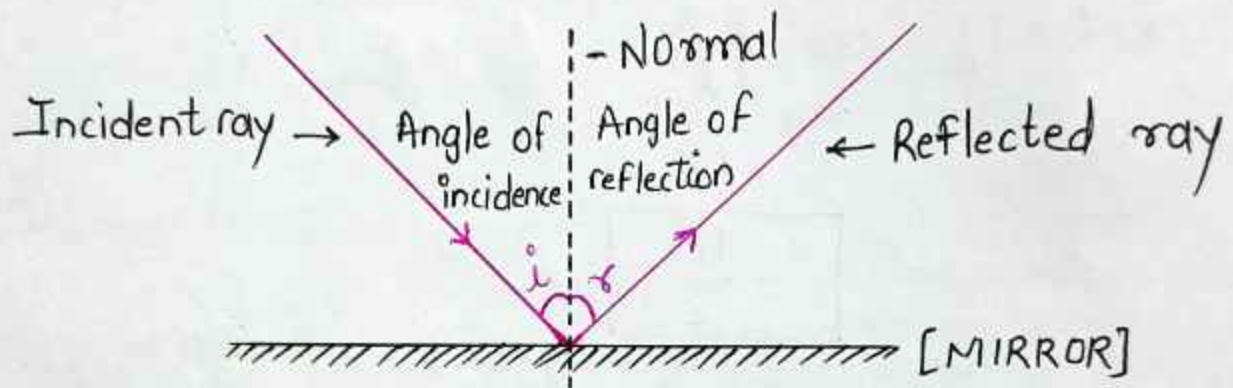
- * PHOTOELECTRIC EFFECTS

- * ABSORPTION OF LIGHT BY THE SUBSTANCE

- * BLACK BODY RADIATION ETC.

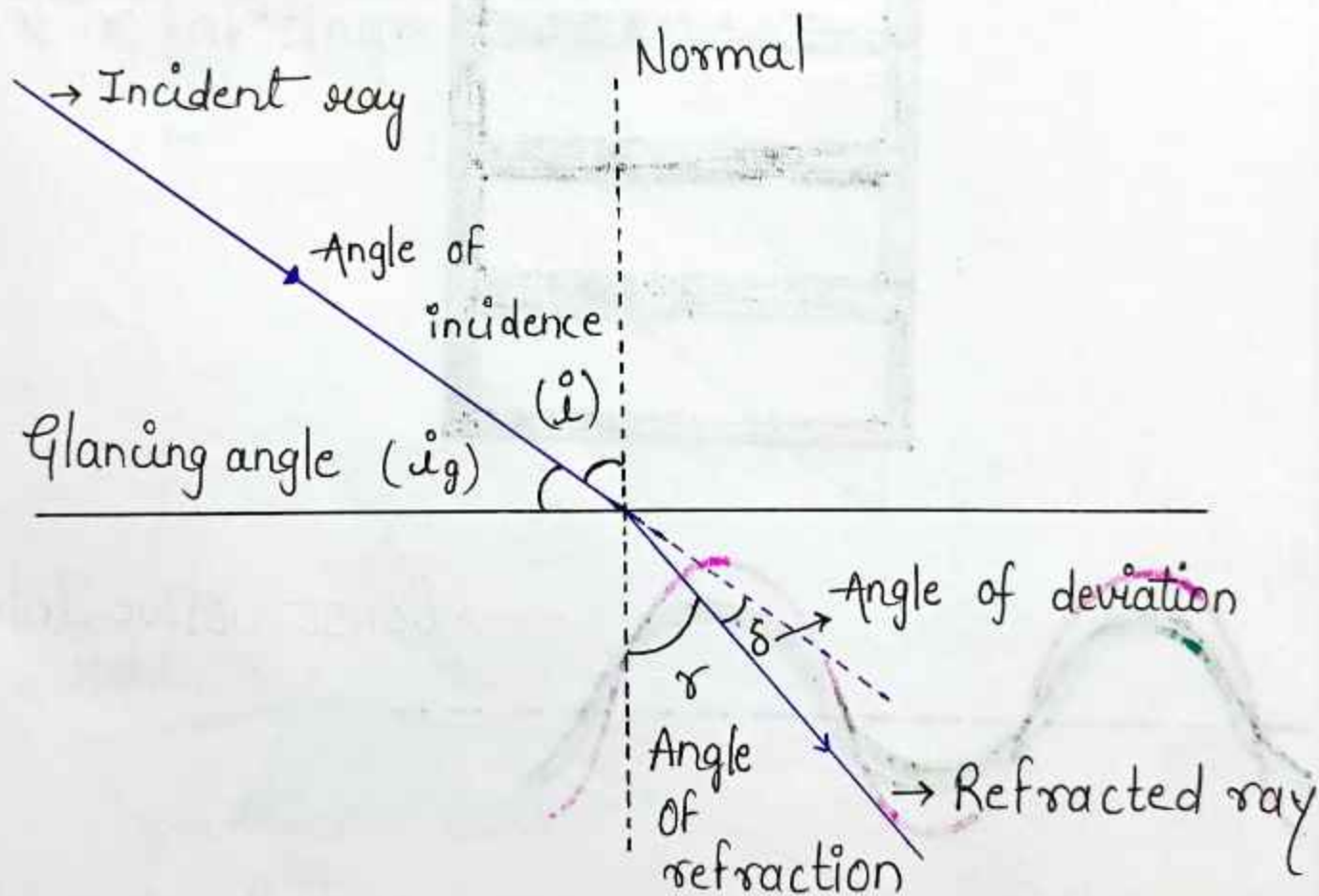
* REFLECTION *

"The bouncing back of light in the same medium by any polished surface is known as Reflection of light."

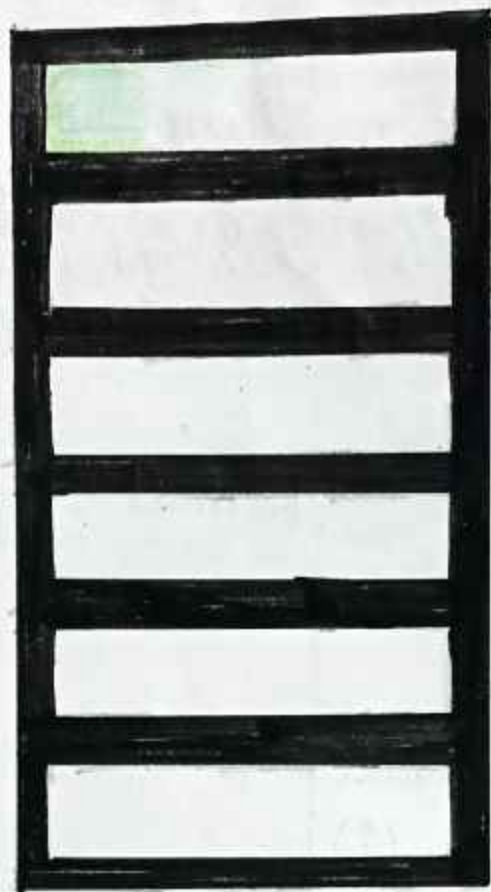


* REFRACTION *

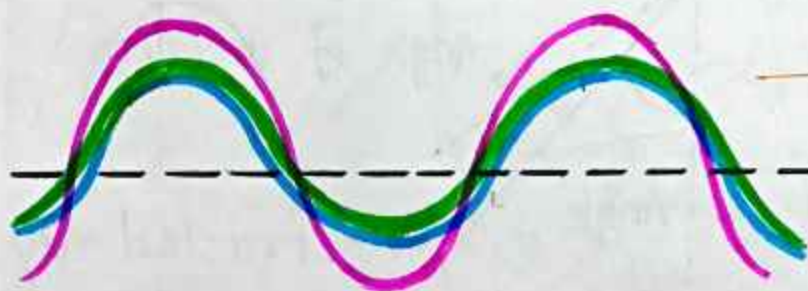
"When light rays are coming from medium to another medium of different optical densities then they deviate from their original path. This phenomenon of light is known as Refraction of light."



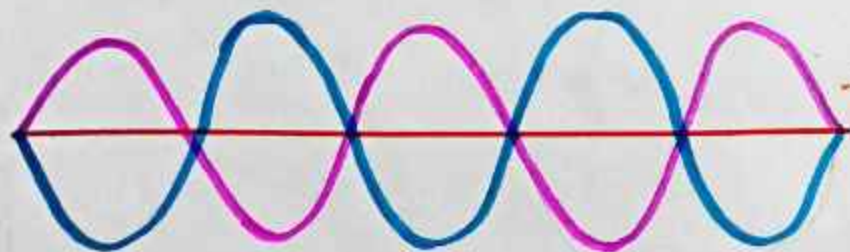
* INTERFERENCE *



Interference
pattern on screen

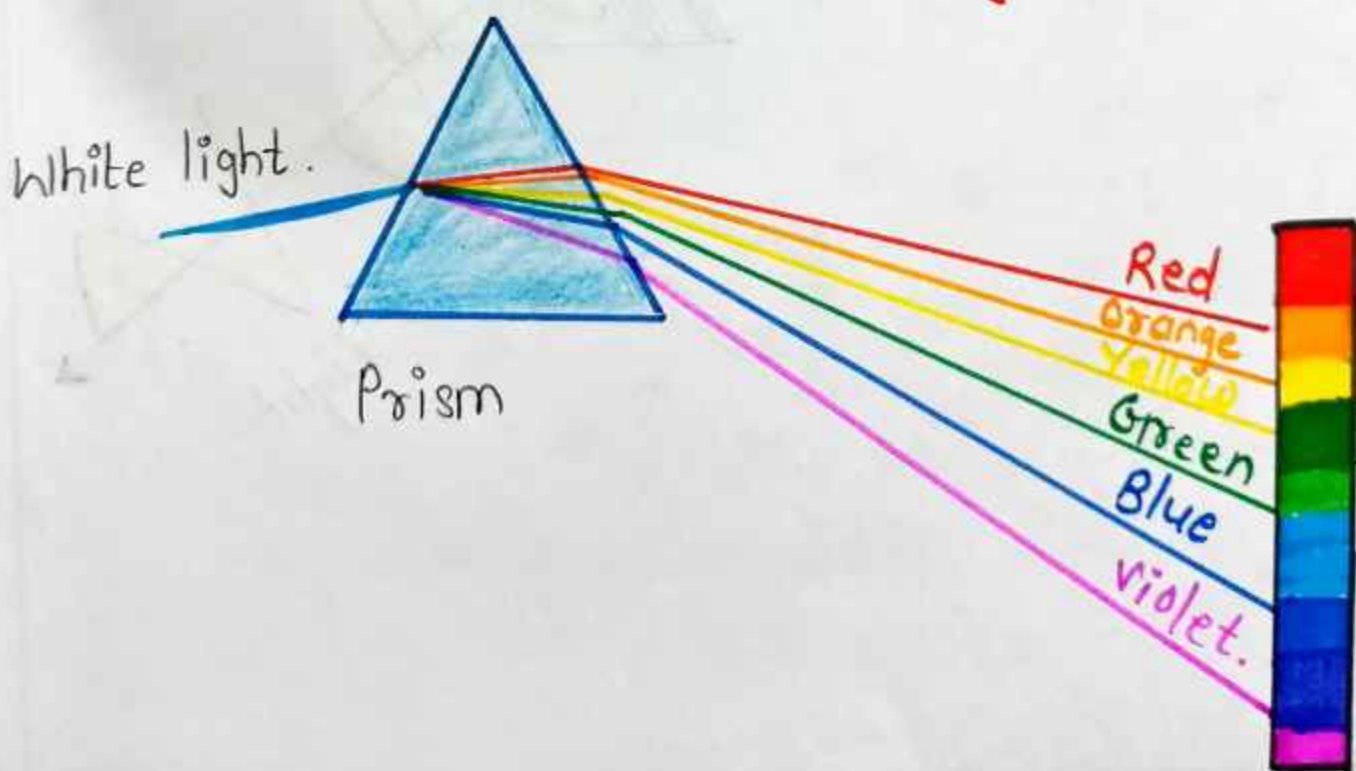
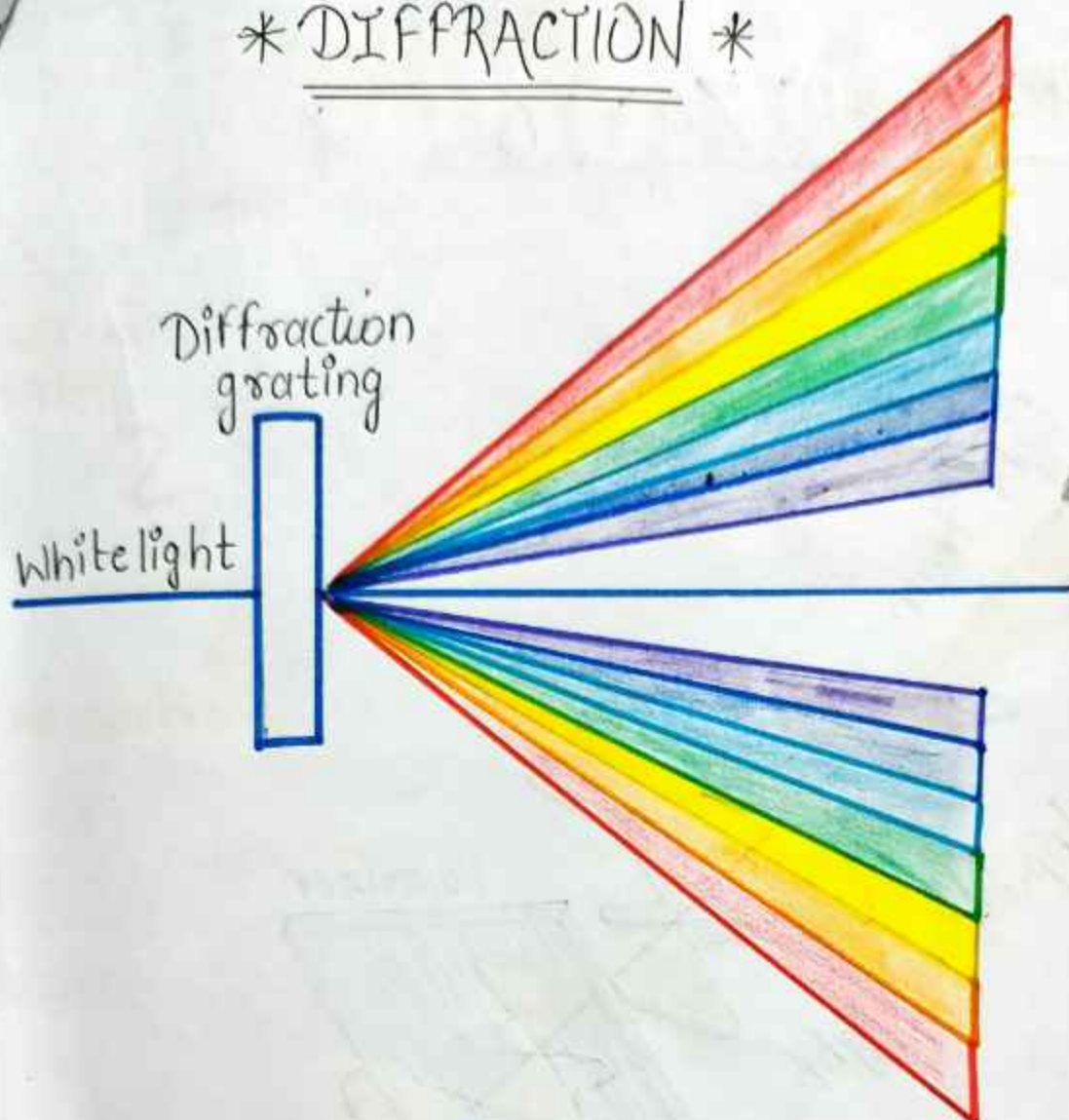


→ Constructive Interference

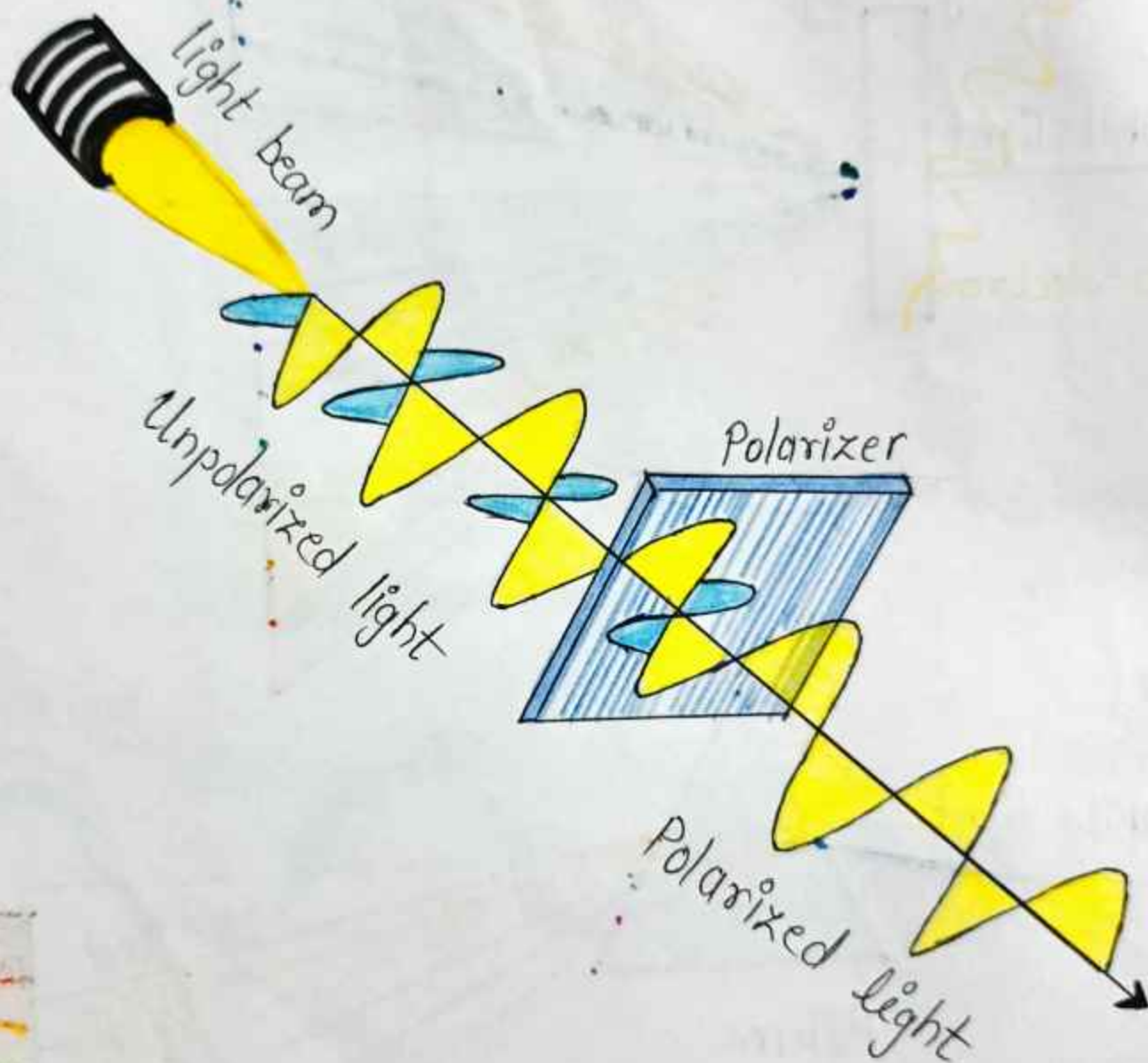


→ Destructive Interference

* DIFFRACTION *

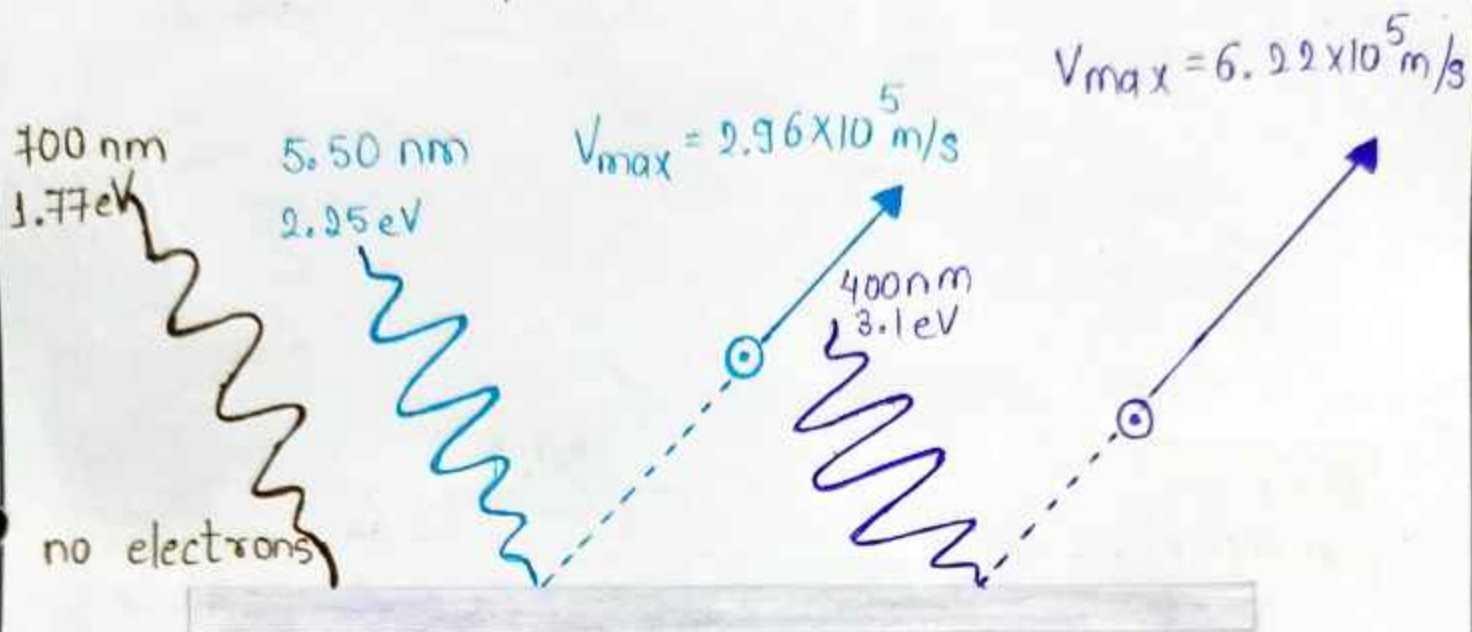


* POLARIZATION *



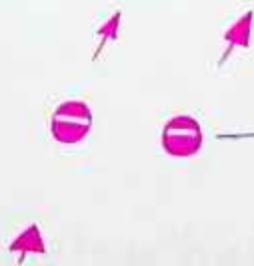
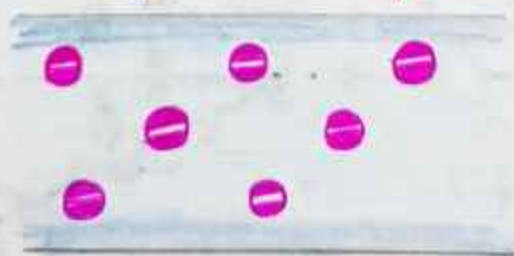
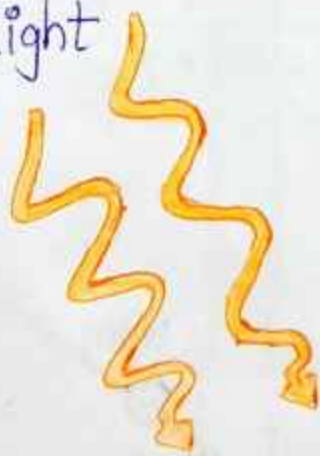
* PHOTOELECTRIC EFFECT *

$$E_{\text{photon}} = h\nu$$



Potassium - 2.0 eV needed to eject electron

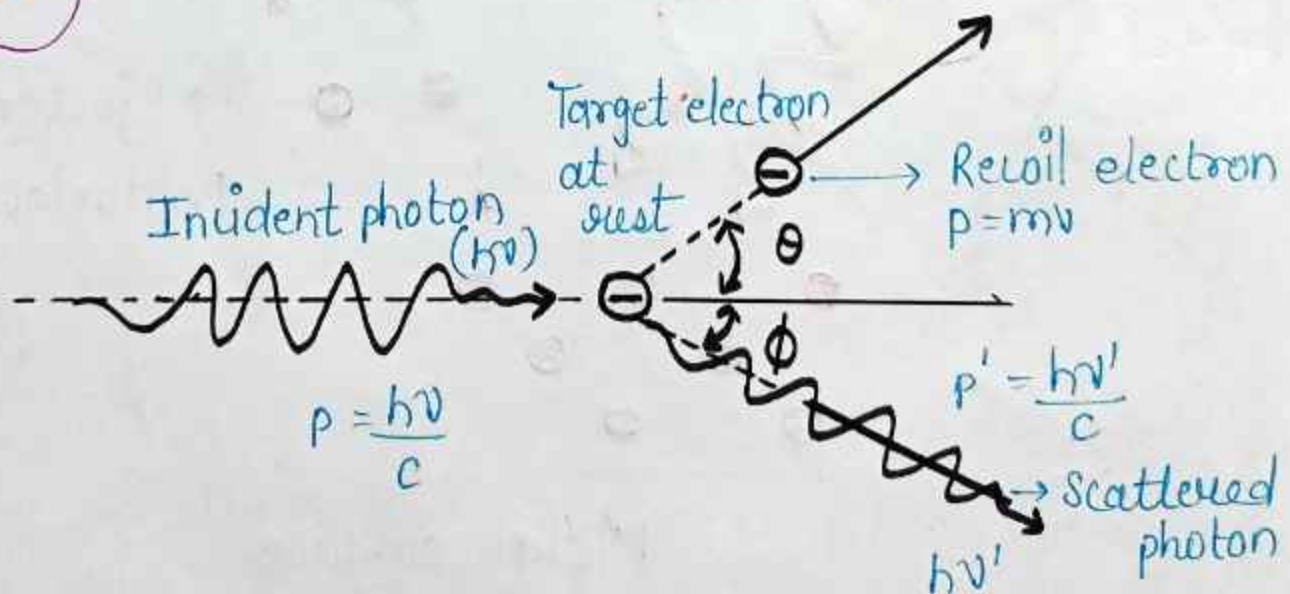
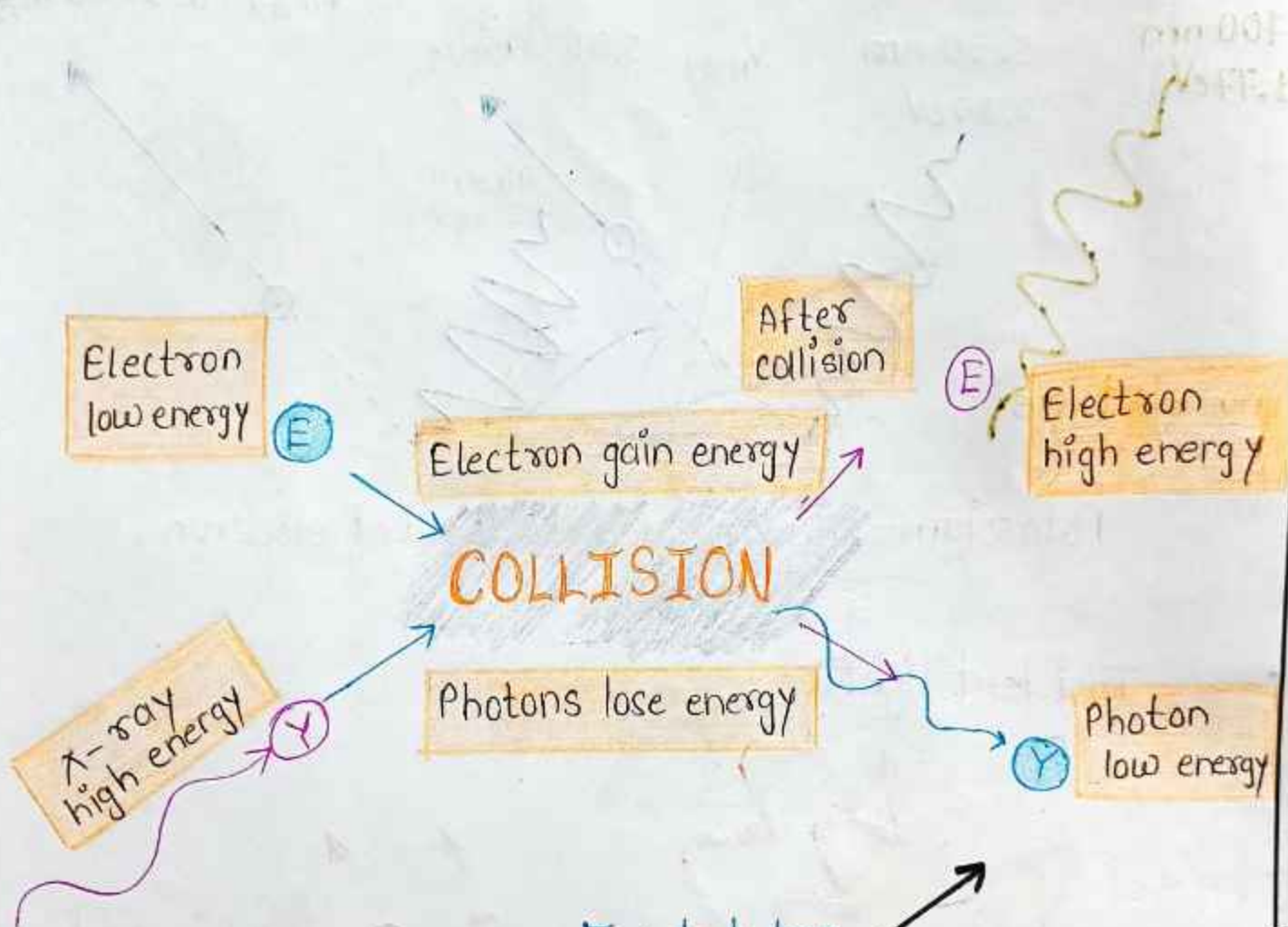
Incident light



Ejected Photoelectron

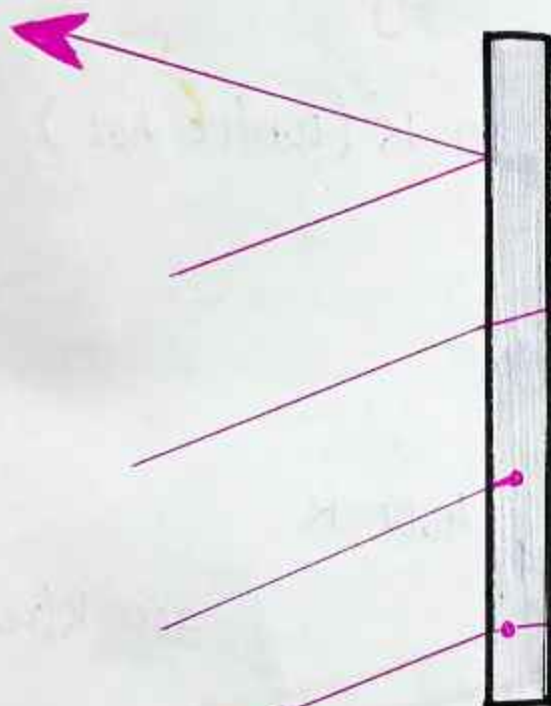
Metal surface

* COMPTON EFFECTS *



* ABSORPTION OF LIGHT *

Reflection

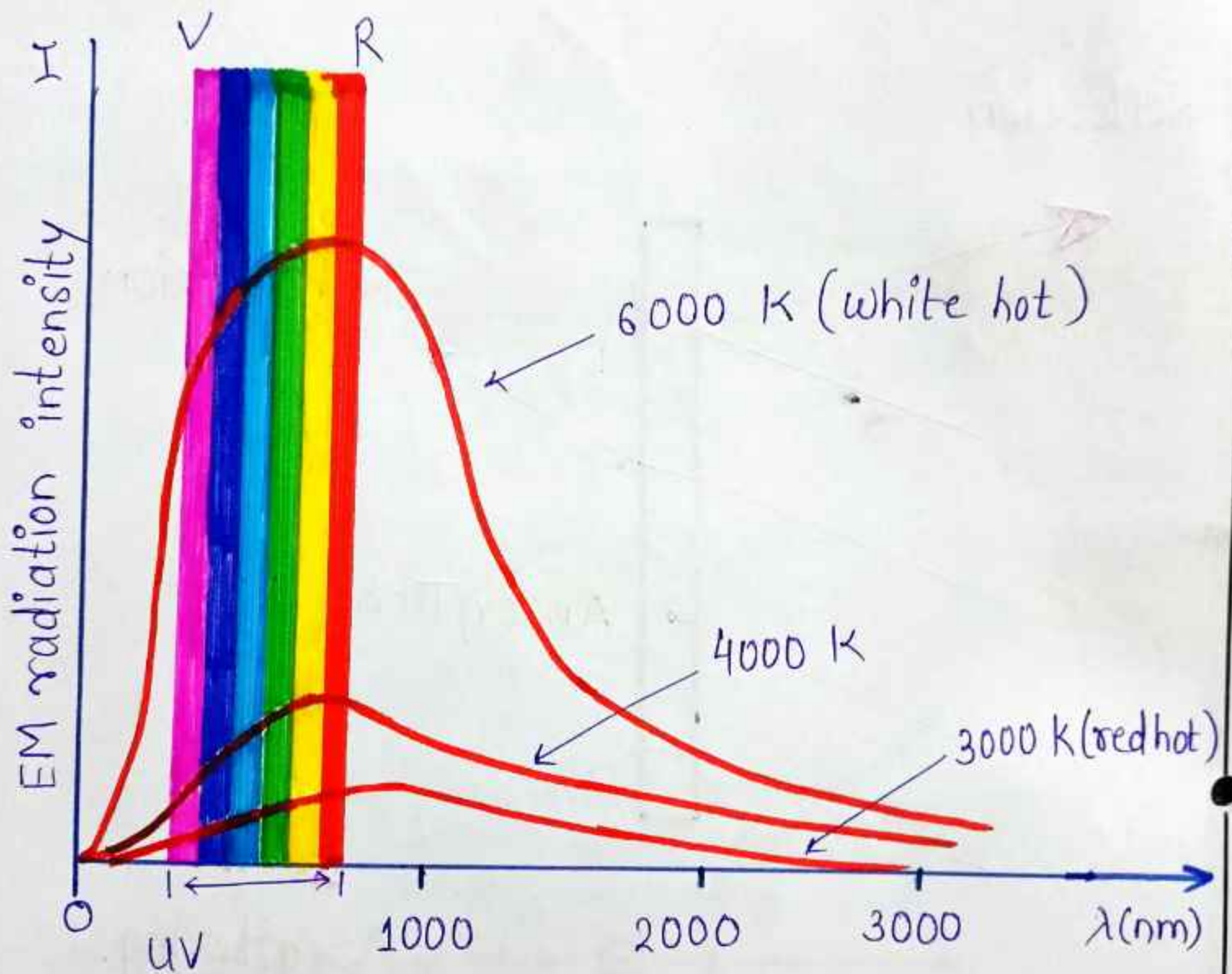


Transmission

Absorption

Scattering.

*BLACK BODY RADIATION *



INTRODUCTION OF QUANTUM PHYSICS.

The optical phenomenon such as Reflection, refraction, Interference, Diffraction and polarization. explain by the wave theory but wave theory of light failed to explain photoelectric effect, Compton effect absorption of light and initial of radiation by the substance. These phenomenon easily explain by Planck's Quantum Theory.

According to which the emission and absorption of light is not continuous but it is discrete (light is in the form of small bundles and packets of energy which are called photon or Quanta).

Thus, the photon theory and wave theory are required to explain on the light phenomenon i.e. the light has the dual nature i.e. particle nature & wave nature.

A photon of frequency ν , energy $E = h\nu$ and momentum $h\nu/c$, where h is the Planck's $\hbar$ constant. Its value is 6.62×10^{-34} J/Sec and c is the speed of light. Its speed value is 3×10^8 m/s. Photon is energy particle.

* QUANTA *

It is a packet of energy its energy

$$E = h\nu$$

⇒ PLANCK'S THEORY :-

* Dual Nature of light —

It is the partical nature and wave nature. It is clear that the phenomenon of photoelectric effect, compton effect, absorption of light explain by partical nature of light but the wave nature is explain-ed by the phenomenon of reflection, refraction Interference and Polarisation established light in the form of a continuous wave.

REFERENCES:- Basic Engg. Physics, by Hovmeet

SUMMARY:- Light has dual nature as well as Microparticle Such as particle and wave and photon Energy $E = h\nu$

TOPIC:- Schrodinger wave equation,

OBJECTIVES:- To Derive the Schrodinger time and time independent wave eqn

OUTCOME:- Student will be able to derive Schrodinger wave eqn

Time dependent Schrodinger wave eqn:-

$$\psi = ae^{-i(Et - Px)/\hbar} \quad \text{--- (i)}$$

Consider a particle of mass m moving in the x direction having potential energy $V(x)$ and total energy E . Matter associates a particle nature of light. taking (i)

wave function

$$\psi = ae^{-i(Et - Px)/\hbar}$$

By partially diff'n w.r.t. t --- (i)

$$\frac{\partial \psi}{\partial t} = ae^{-i(Et - Px)/\hbar}$$

$$\left[\frac{-i}{\hbar} E \cdot 1 \right]$$

$$\frac{\partial \psi}{\partial t} = \psi \left(\frac{-iE}{\hbar} \right)$$

$$\Rightarrow \frac{\partial \psi}{\partial t} = \frac{-i}{\hbar} E \psi$$

$$E \psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$= \frac{-i\hbar \partial \psi}{i^2 \partial t}$$

$$E \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\boxed{E = \frac{i\hbar \partial}{\partial t}} \Rightarrow E = i\hbar \frac{\partial}{\partial t}$$

$$\psi = ae^{-i(Et - Px)/\hbar}$$

diffn with respect to x — (i).

$$\frac{\partial \psi}{\partial x} = ae^{-i(Et - Px)/\hbar}$$

$$\frac{\partial \psi}{\partial x} = \psi \left(\frac{iP}{\hbar} \right)$$

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P \psi$$

$$P \psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}$$

$$= \frac{i\hbar}{i^2} \frac{\partial \psi}{\partial x}$$

$$P \psi = -i\hbar \frac{\partial \psi}{\partial x}$$

$$P = -i\hbar \frac{\partial}{\partial x}$$

Total Energy - $E = K.E + P.E$

$$E = P^2/2m + V \quad \text{--- (3)}$$

From eqn (1), (2), (3)

$$\Rightarrow i\hbar \frac{\partial}{\partial t} = \frac{1}{2m} \left(i\hbar \frac{\partial}{\partial x} \right)^2 + V$$

$$i\hbar \frac{\partial}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial}{\partial x} \cdot \frac{\partial}{\partial x} + V$$

$$\Rightarrow i\hbar \frac{\partial}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$$

Taking both side wave function.

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} + V(\psi)$$

In 3D $\Rightarrow \frac{-\hbar^2}{2m} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] \psi + V(\psi)$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V \psi$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

For Free particle.

$$i\hbar \frac{\partial}{\partial t} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \quad (1D) \quad V=0$$

$$\boxed{i\hbar \frac{\partial}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2} \quad (3D)$$

* Time Independent Schrodinger wave eqn.

Consider a particle of mass m moving along to the +ve x -direction with small v velocity have momentum P , Energy E .

W.K.T. Wave Ψ^n .

$$\Psi = ae^{-i(Et - Px)/\hbar}$$

Diffⁿ with r.t. x .

$$\frac{\partial \Psi}{\partial x} = ae^{-i(Et - Px)/\hbar} \cdot \frac{Pi}{\hbar}$$

$$\frac{\partial \Psi}{\partial x} = \Psi \cdot \frac{Pi}{\hbar}$$

$$P(\Psi) = \frac{\hbar}{i} \frac{\partial \Psi}{\partial x}$$

$$\boxed{P = -i\hbar \frac{\partial}{\partial x}} \quad -(i).$$

$$E = \frac{P^2}{2m} + V$$

$$= \frac{1}{2m} \left(-i\hbar \frac{\partial}{\partial x} \right)^2 + V$$

$$= \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} + V$$

$$E\Psi = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

$$-\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} + E\psi - V\psi = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0} \quad (1D)$$

For 3D

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\} \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0.$$

$$\Rightarrow \boxed{\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0} \quad (3D)$$

where

For free particle $V=0$.

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2} E \psi = 0}$$

REFERENCES:- Claire B.Sc. M. Goyal.

SUMMARY:- Schrodinger wave eqⁿ

Time dependent $= i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$

Time independent $= \nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$

TOPIC :- Solution of Schrodinger eq in 1D.

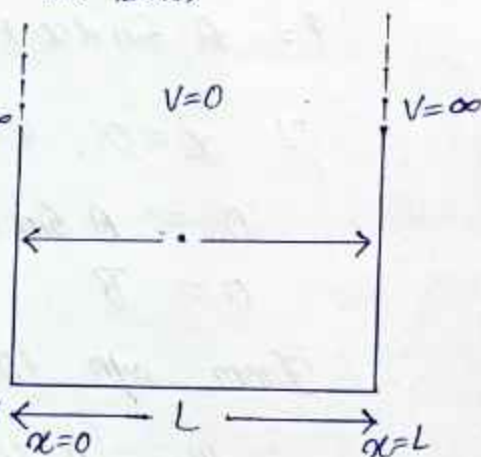
OBJECTIVES:- To determine the Energy Eigen value and Eigen function

OUTCOME:- Stu will be able to find Energy value when particle in finite region.

Case-1:- $V=0$, $0 < x < L$ (inside the box)

⇒ Particle in a Rigid one dimensional $V=\infty$ Potential box.

i). Particle can freely move inside the box no force act on the particle.
i.e. potential $V=0$ $0 < x < L$.



ii). Outside the box Particle not surrounded by the box there Potential $V=\infty$
boundary condition. $V=\infty$, $x \leq 0$ $x \geq L$

→ Consider a particle moving inside a box along to the x -direction. The particle jumped back and forth b/w the walls of the box.

L is the width of the box. And potential energy ' V '.

Particle can't exist outside the box.
So its wave fun $\psi=0$ $x \leq 0$, $x \geq L$

Now, Schrodinger wave independent eqn.

For free particle with in box.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} EV = 0 \quad \text{--- (1)}$$

$$\text{Let } \alpha^2 = \frac{2mE}{\hbar^2}$$

From eqⁿ (i).

$$\frac{d^2 \psi}{dx^2} + \alpha^2 \psi = 0.$$

$$\psi = A \sin \alpha x + B \cos \alpha x \quad \text{--- (2)}$$

$$\text{At } x=0, \psi=0$$

$$0 = A \sin 0 + B \cos 0$$

$$0 = B.$$

From eqⁿ (2)

$$\psi = A \sin \alpha x \quad \text{--- (3)}$$

$$x = L, \psi = 0$$

$$0 = A \sin \alpha L$$

$$\Rightarrow \sin L = 0, A \neq 0$$

$$\Rightarrow \sin \alpha L = \sin (n\pi)$$

$$\alpha L = (n\pi)$$

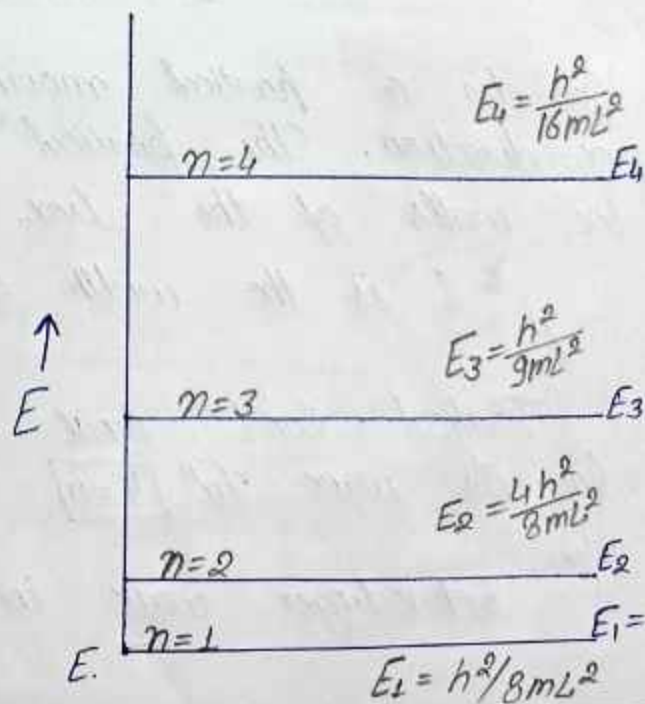
$$\alpha = \frac{n\pi}{L}$$

$$\psi = A \sin \alpha x$$

$$\psi = A \sin \left(\frac{n\pi x}{L} \right)$$

$$\alpha = \frac{n\pi}{L}$$

$$\alpha^2 = \frac{2mE}{\hbar^2} \quad \text{--- (A)}$$



$$E = \frac{\alpha^2 \hbar^2}{2m}$$

$$E = \frac{\left(\frac{n\pi}{L}\right)^2 \hbar^2}{2m}$$

$$E = \frac{n^2 \pi^2}{L^2} \times \frac{\hbar^2}{4\pi^2} \times \frac{1}{2m}$$

$$E = \frac{n^2 \hbar^2}{2mL^2}$$

$$\boxed{E_n = \frac{n^2 \hbar^2}{8mL^2}} \quad \text{--- (B)}$$

$$n=1 \quad \boxed{E_1 = \frac{\hbar^2}{8mL^2}}$$

REFERENCES:- R. P. Feynman Unified. B.Sc.

SUMMARY:-

$$E_n = \frac{n^2 h^2}{8mL^2} \quad \text{Energy Eigen Value}$$

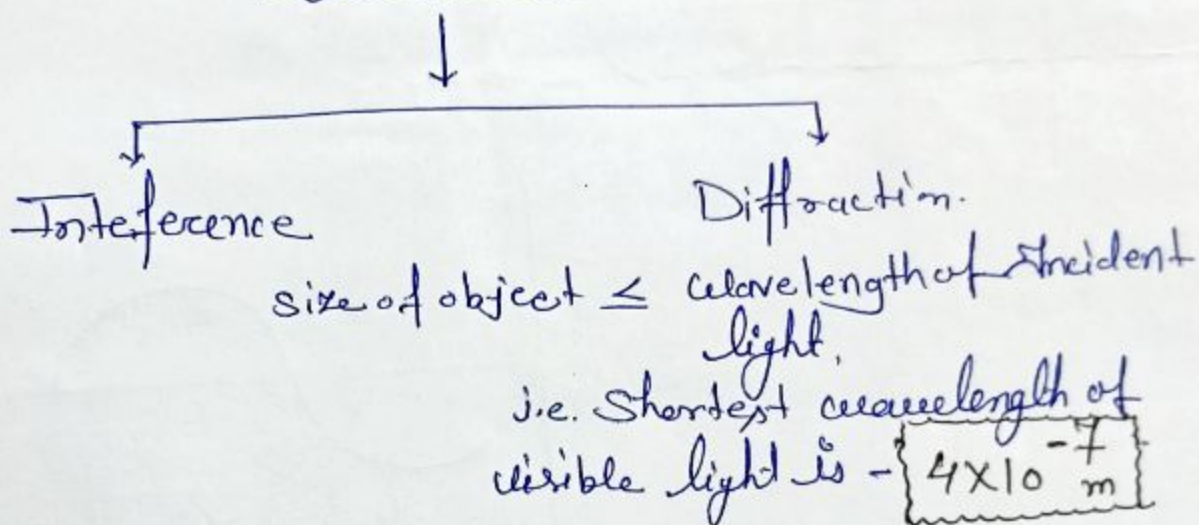
$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

TOPIC:- Principle of Superposition

OBJECTIVES:- To understand the principle of superposition

OUTCOME:- Student will be able to explain as well as understand the principle of superposition

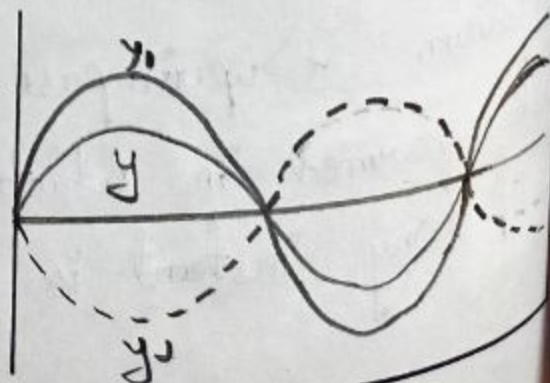
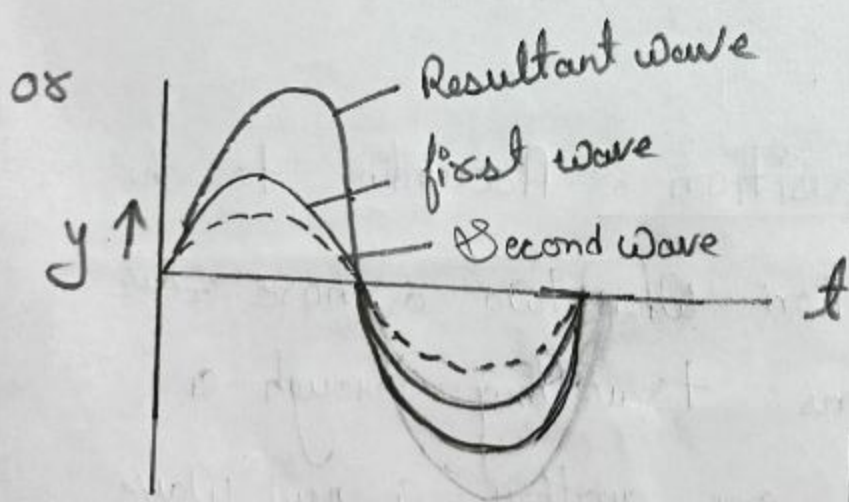
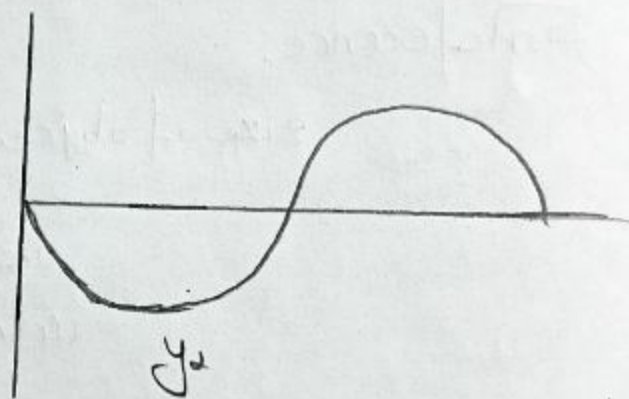
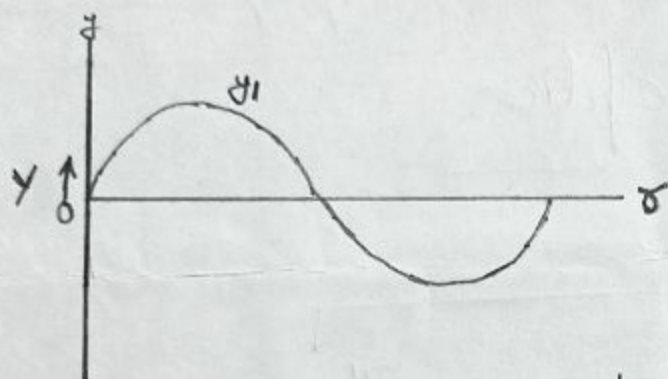
Wave-optics



principle of Superposition :- According to the principle of Superposition When two or more waves then two motions travelling through a Medium Superimpose one another, a new wave is formed in which resultant displacement is of any instant is equal to the Sum of

displacement due to individual waves at the inst.

$$\text{i.e. } \vec{y} = \vec{y}_1 + \vec{y}_2 - - - - + \vec{y}_n$$



TOPIC :- Nature of light

OBJECTIVES:- To Understand the nature of light

OUTCOME:- ~~She~~ will be able to understand the nature of light

Nature of light In the General.

In General:-

Light is a form of energy, when light after reflection from an object, enters our eye, the object is seen.

"Actually the light produces the sensation of elusion of retina of our eyes.

"light is a form of energy which produces a sense of elusion on the retina of our eye.

Now,
Some phenomenon of light

1) Rectilinear Propagation of light:-

Light travels in a straight line, this evidence is the formation of shadow of an opaque object in the path of light.

Similarly, formation of solar calipse when moon comes in between sun and earth.

formation of linear eclipse when earth comes in between Sun and earth.

Solar eclipse (सूर्यगृहण) corpuscles (कणिकाएं)

i) Reflection, (ii) Refraction, (iii) Interference, (iv) Photo electric effect, (v) Diffraction, (vi) Polarization, (vii) Dispersion, (viii) Compton effect (ix) Black body Radiation etc.

→ Rectilinear propagation of light:-

Which is explain by Newton's Corpuscular theory

According to which the corpuscles are of negligible mass. and they travel with a tremendously high speed, so they experience a negligible force of gravity — As a result the corpuscles travel in a straight line.

TOPIC :- Huygen's wave theory, wave front, secondary.

OBJECTIVES:- To Explain and Define Huygen's ^{wavelets} theory.

OUTCOME:- Stu will be able to Explain Huygen wave theory

Now,

Huygen's Wave theory:-

- 1) light travels in form of waves,
- 2) Initially the light waves were assumed to be longitudinal, But later on while explaining the phenomenon of polarisation, the waves were considered to be transverse.
- (iii) The waves travel in all direction with the speed of light.

The waves of light of different with the speed
Colours have different frequencies.

For the propagation of waves, imaginary medium called ether was considered and it was assumed that the ether per waves in all space. It is massless and homogenous medium of very high electricity and Low density.

But, This theory could not explain rectilinear propagation of light. and Explain - Reflection, Refraction, interference, diffraction and polarization of light.

Later on Fresnel Found:- that the rectilinear propagation of light is not perfectly true.

Light bends at the corners of an obstacle or aperture is of the order, when the size of the obstacle or aperture is of the order of wavelength of light.

Then explaining refraction with this theory, it is proved that speed of light is more in a rarer medium and less in denser medium.

To explain the optical phenomenon by Huygen's wave theory, a wavefront is assumed and for it Huygen's propounded a principle of secondary wavelets.

Wavefront and Rays:-

Wavefront:- A surface drawn at any instant in the medium affected by the waves originated from a source on which each particle vibrates in same phase. and if wave propagates in a medium, at any instant the locus of all the particles of the medium which are being disturbed at the same time and are in the same phase of vibration, is called wavefront.

The transfer of energy at any point of the

Wavefront is in direction normal to the wavefront at that point.

This direction of energy transfer is called the ray. This a ray is the line drawn normal to the wavefront.

Types of Wavefront

- i) Plane Wavefront
- ii) Spherical wavefront
- iii) Cylinder wavefront

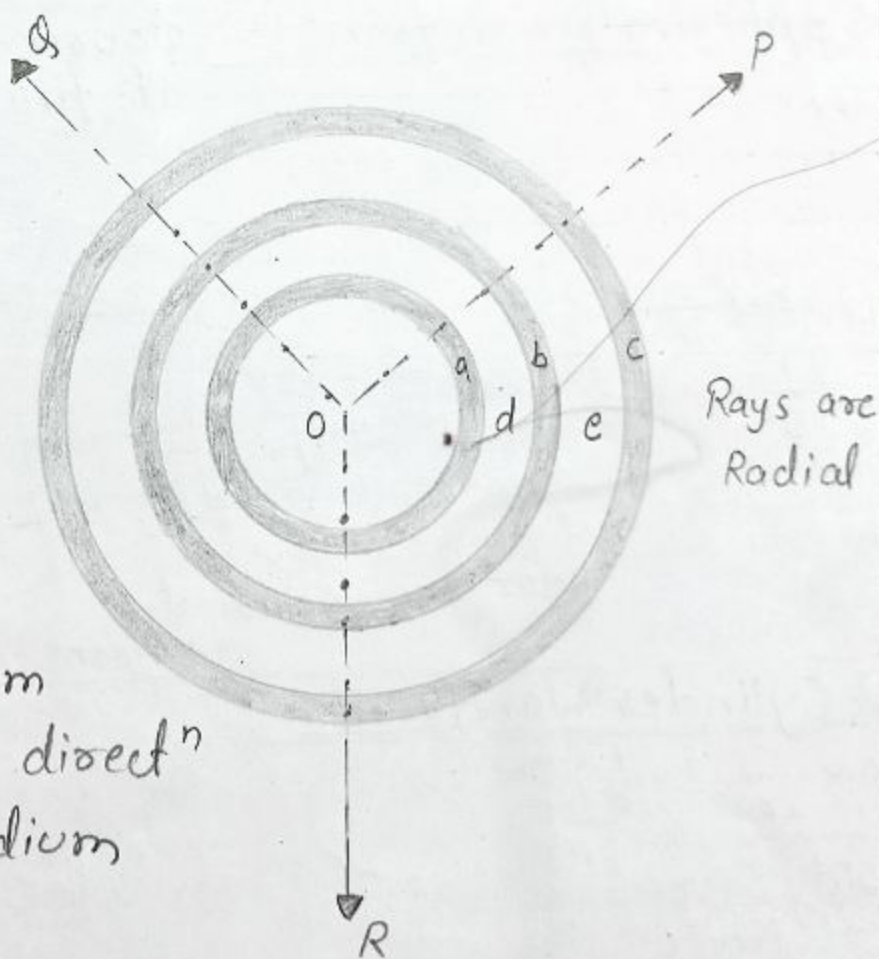


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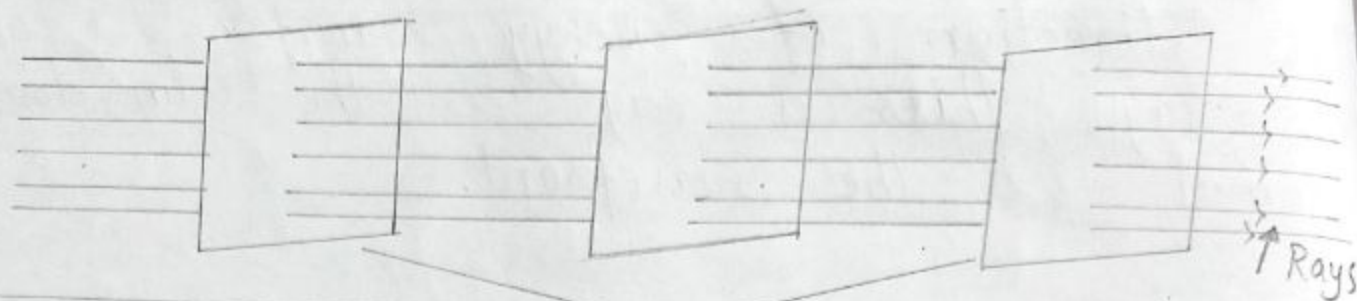
OP, OQ, OR - Ray
Which is $\perp$ to
Wavefront

a, b, c $\rightarrow$ Crest
d, e $\rightarrow$ Troughs

- Particle in medium
vibrate in same directⁿ
and same medium



1) Plane Wavefront $\rightarrow$ Source of light at ∞ .



Plane wavefront || ray
then its
amplitude and
Intensity constant

Plane - Wavefront

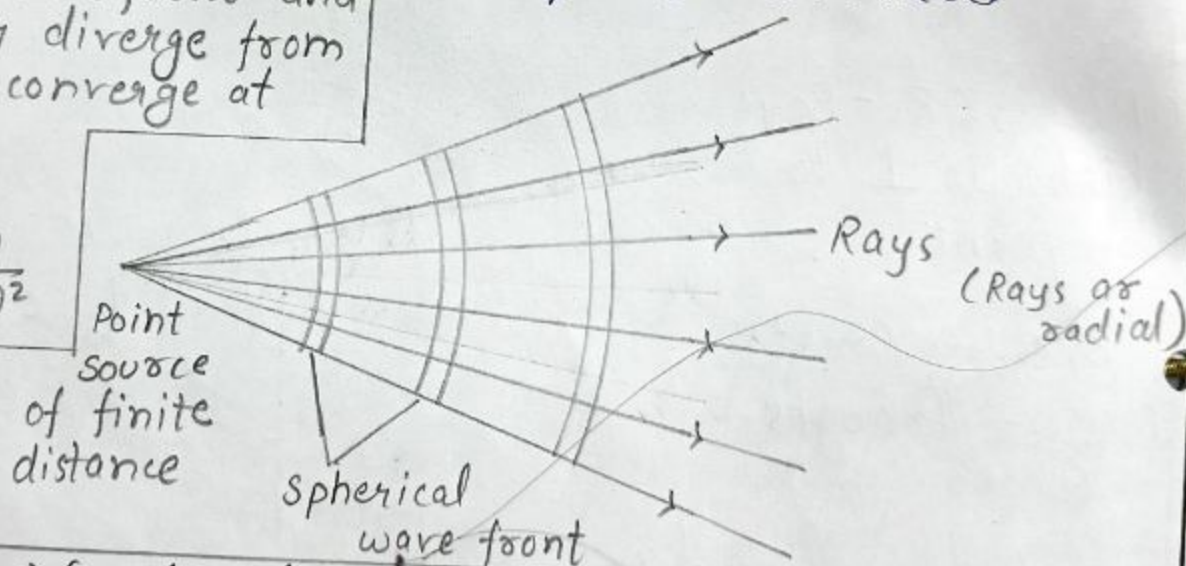
if a plane surface
drawn at any
instant to the
directⁿ of
wave propagation

2) Spherical Wavefront:- Source of light is point
at finite distance.

spherical wavefront and
rays either diverge from
a point or converge at
a point.

i.e. $A \propto \frac{1}{r}$,

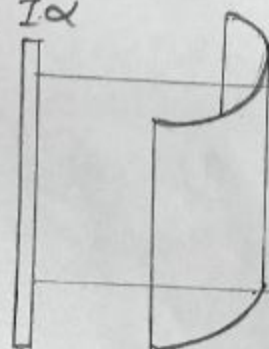
$I \propto \frac{1}{r^2}$, $I \propto \frac{1}{A^2}$



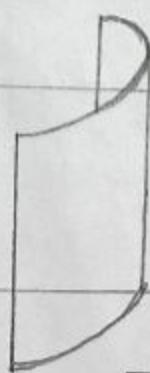
3) Cylindrical Wavefront:-

$A \propto \frac{1}{\sqrt{r}}$, $I \propto \frac{1}{r}$, $I \propto$

slit source
at finite
distance



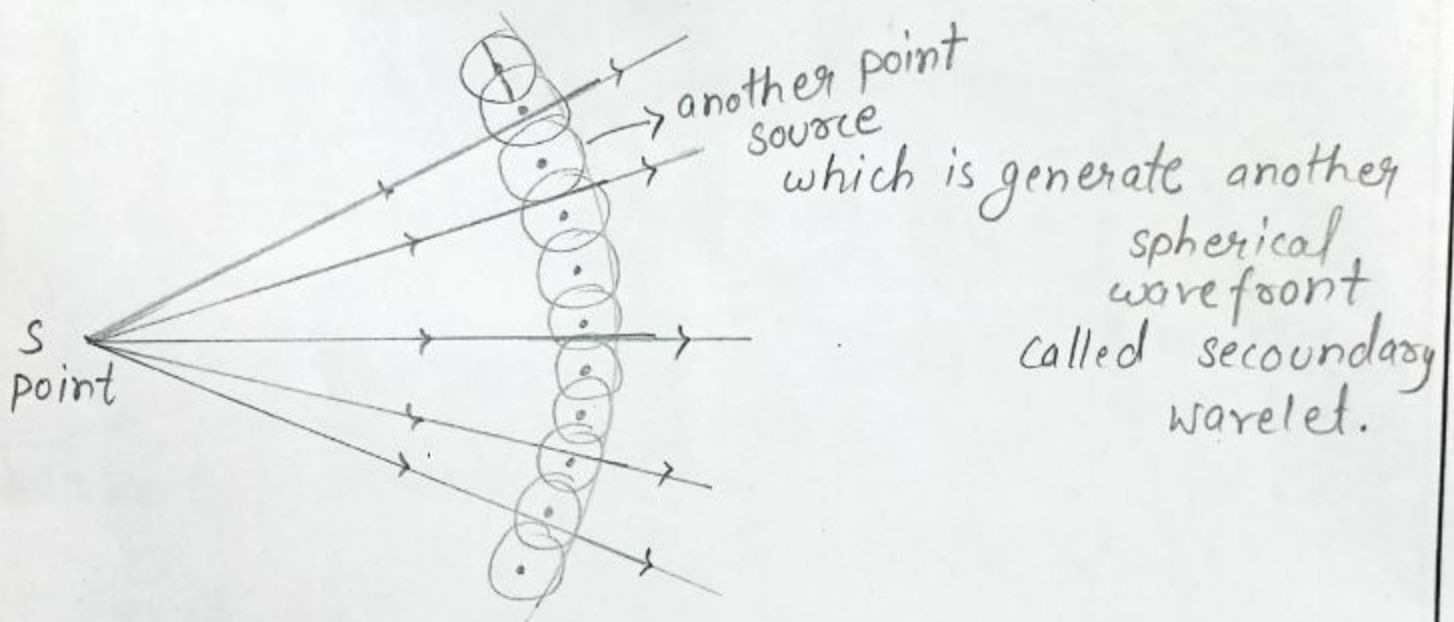
r is distance
from the source



Cylindrical wavefront

Huygen's Principle of Secondary wavelets:-

Every point of the medium on a given wavefront is a centre of originating new disturbance. This new disturbance is called the secondary wavefront.



PAGE NO

TOPIC :- Interference ;

OBJECTIVES:- To understand the interference.

OUTCOME:- Str. will be able to understand interference.

Interference :- When two waves of light nearly equal amplitude but same frequency and wave length intensity of light is different at different between of the Medium,

At some point the intensity of light is maxi. While at some point the intensity is 0 at some other point of light is minimum.

Thus the format of maxi intensity at some point and minimum intensity 0 at some other point.

By the 2 identical light waves of same frequency & nearly same amplitude travelling in same direction is called interference of light.

Constructive and destructive interference :-

Consider two wave have nearly Same amplitude
Same frequency Same wave length propagate
Same directⁿ in Same Medium that the displacement
eqⁿ for both wave.
is .

$$y_1 = a_1 \sin \omega t \text{ --- (i)}$$

$$y_2 = a_2 \sin (\omega t - \phi) \text{ --- (ii)}$$

A/c to principle of Superposition

$$y = y_1 + y_2$$

$$y = a_1 \sin \omega t + a_2 \sin (\omega t - \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= \sin \omega t (a_1 + a_2 \cos \phi) + a_2 \cos \omega t \sin \phi \text{ --- (3)}$$

let

$$a_1 + a_2 \cos \phi = A \cos \theta \text{ --- (A)}$$

$$a_2 \sin \phi = A \sin \theta \text{ --- (B)}$$

Now put from eqⁿ - (3) of eqⁿ A & B

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

$$y = A [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$[y = A \sin(\omega t + \phi)] - \text{Displacement}$$

from eqⁿ A and B Squaring both sides and adding.

then,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

Intensity, Now it is directly proportional to the A^2

$$I \propto A^2$$

$$I \propto (a_1^2 + a_2^2 + 2a_1a_2 \cos \phi)$$

$$I = K (")$$

for constructive, then $\pi = 0, 2\pi, 4\pi, \dots, 2n\pi$
 $n = 0, 1, 2, 3, \dots$

Path difference.

$$\Delta = \frac{\lambda}{2\pi} \times \text{phase difference}$$

$$= 0, 1, 2, 3, \dots, n-1$$

$$[\cos \phi = 1]$$

$$A_{\max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$$

$$A_{\max} = \sqrt{(a_1 + a_2)^2}$$

$$A_{\max} = (a_1 + a_2)^2$$

$$A_{\max} = a_1 + a_2$$

$$I_{\max} \propto A_{\max}^2$$

$$I_{\max} = k A_{\max}^2$$

$$= k (a_1 + a_2)^2$$

$$= k a_1^2 + k a_2^2 + 2a_1a_2$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

for destructive

$$\cos \phi = -1$$

$$\phi = \pi, 3\pi, 5\pi, \dots, (2n-1)\pi$$

$$n = 1, 2, 3, \dots$$

Path difference

$$\Delta = \frac{\lambda}{2\pi} \times \text{phase difference}$$

$$= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots, \frac{(2n-1)\lambda}{2}$$

for min :-

$$A_{\min} = \sqrt{a_1^2 + a_2^2 - 2a_1a_2}$$

$$A_{\min} = \sqrt{(a_1 - a_2)^2} = a_1 - a_2$$

$$I_{\min} \propto A_{\min}^2$$

$$I_{\min} = k A_{\min}^2$$

$$= k (a_1 - a_2)^2$$

$$= k a_1^2 - k a_2^2 - k a_1 a_2$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Case II :-

When both wave have Same Amplitude
 $a_1 = a_2 = a$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi} \quad \text{--- (C)}$$

$$A = \sqrt{2a^2 + 2a^2 \cos \phi}$$

$$A = \sqrt{2a^2 (1 + \cos \phi)} = \sqrt{2a^2 + 2\cos^2 \phi/2}$$

$$A = 2a \cos \phi/2$$

Intensity :-

$$I = k 4 a^2 \cos^2 \phi/2$$

$$A_{\max} = 2a, \quad I_{\max} = 4k a^2$$

$$A_{\min} = I_{\min} = 0.$$

TOPIC:- Type of Interference, Young's Experiment,

OBJECTIVES:-

To identify the phenomenon called Interference in which two coherent light wave sources

OUTCOME:-

Students will be able to identify coherent sources as well as interference.

Interference

due to division of wavefront

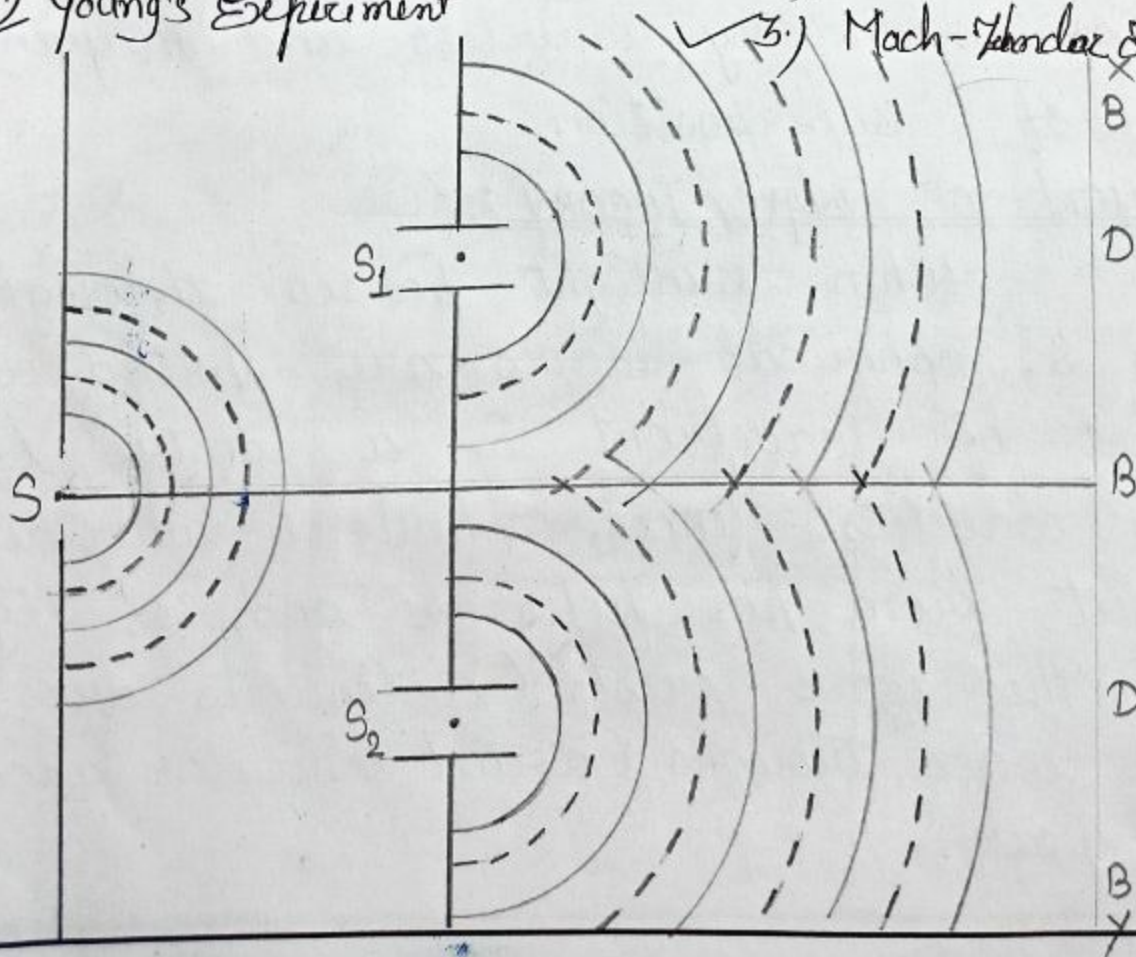
↓ ex.

- 1) Fresnel biprism.
- 2) Lloyd's Mirror
- 3) Young's Experiment

due to division of Amplitude

↓ ex

- ✓ 1) Newton's Ring Experi.
- ✓ 2) Michelson's interferometer
- ✓ 3) Mach-Zehnder Interferometer



In 1801, Thomas Young first demonstrated experimentally the phenomenon of Interference of light. Youngs allowed sunlight to pass through a hole and then at some distance through two pin holes S_1 and S_2 . When light was recieved on a screen an uneven distribution of light intensity was observed on the screen. A number of alternate dark and bright spots were seen on the screen.

* PRINCIPLE :-

It is based on the Huygen's principle of secondary wavelets and Huygen's principle of superposition.

* EXPLANATION OF WAVE THEORY :-

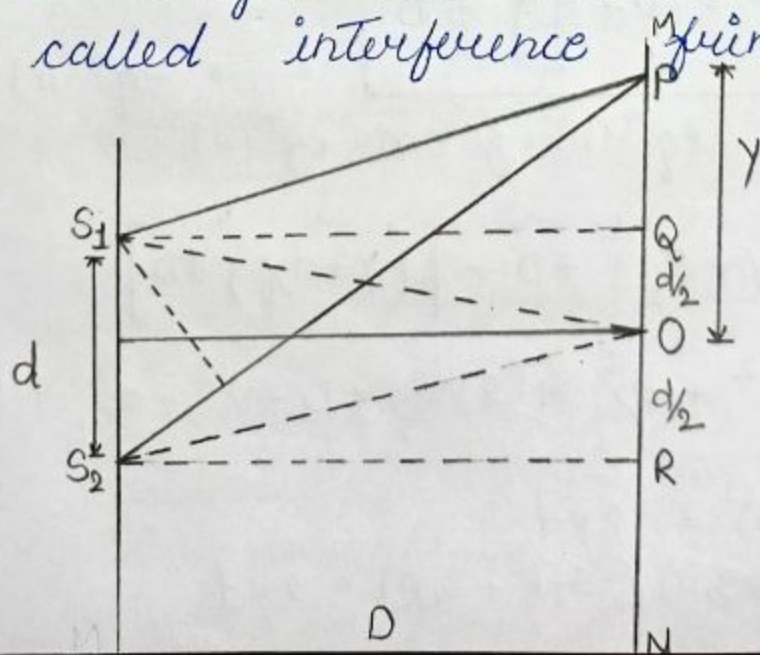
When sunlight passes through pin holes S , spherical waves are spread out. Each point of wavefront is a centre of secondary wavelets. Hence, spherical wave also spread out from pin holes S_1 and S_2 . Fig represents the wave crests (continuous arcs) while the wave troughs (dotted circular arcs) in each wave.

At point where a crest due to one wave falls on the crest due to another wave (or trough on a trough), the resultant amplitude is the sum of the amplitude due to each wave separately.

As the intensity is proportional to the square of the amplitude $I \propto A$ and hence the resultant intensity at those points is increased. This is known as Constructive Interference.

At points, where a crest due to one wave falls on the trough of the other wave the resultant amplitude is equal to the difference of the amplitude due to separate waves and hence the resultant intensity is minimum. This is known as Destructive Interference.

Thus, on the screen a number of alternate bright and dark regions of equal width called interference fringes are observed.



Let S_1 and S_2 two coherent sources separated by a distance of d and made from a monochromatic source S . Let screen placed at a distance D from the coherent sources. Point O on the screen which is equidistant from S_1 and S_2 . Therefore path difference at point O is zero & intensity at point O is maximum. Let another point on the screen, So,

$$OP = y, \quad PQ = y - \frac{d}{2} \quad \text{and} \quad PR = y + \frac{d}{2}$$

In ΔS_1QP and ΔS_2RP by using Pythagoras theorem, we get

$$\begin{aligned} (S_1P)^2 &= (PQ)^2 + (S_1Q)^2 \\ (S_1P)^2 &= \left(y - \frac{d}{2}\right)^2 + D^2 \end{aligned} \rightarrow \text{eq}^n(i)$$

Similarly,

$$\begin{aligned} (S_2P)^2 &= (PR)^2 + (S_2R)^2 \\ (S_2P)^2 &= \left(y + \frac{d}{2}\right)^2 + D^2 \end{aligned} \rightarrow \text{eq}^n(ii)$$

Subtracting eqⁿ(i) from eqⁿ(ii)

$$\begin{aligned} (S_2P)^2 - (S_1P)^2 &= \left(y + \frac{d}{2}\right)^2 + D^2 - \left[\left(y - \frac{d}{2}\right)^2 + D^2\right] \\ &= y^2 + \frac{d^2}{4} + 2y\frac{d}{2} + D^2 - y^2 - \frac{d^2}{4} + 2y\frac{d}{2} - D^2 \end{aligned}$$

$$\begin{aligned} (S_2P)^2 - (S_1P)^2 &= 2yd \\ (S_2P - S_1P)(S_2P + S_1P) &= 2yd \end{aligned}$$

$$S_2P \approx S_1P \approx D$$

$$(S_2P - S_1P) \approx D = 2yd$$

$$(S_2P - S_1P) = \frac{2yd}{2D}$$

$$S_2P - S_1P = \frac{yd}{D}$$

$$\text{Path difference} = \frac{yd}{D}$$

For P, the centre of a bright fringe
path difference for bright fringe

$$\frac{yd}{D} = n\lambda$$

$$y = \frac{n\lambda D}{d}$$

For point P, centre of dark fringe

$$\frac{yd}{D} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad (2n-1)\frac{\lambda}{2}$$

$$y = \frac{D(2n+1)\lambda}{2d}, \quad y = \frac{D(2n-1)\lambda}{2d}$$

Let, y_n & y_{n+1} the distance of the n^{th} & $(n+1)^{\text{th}}$ bright fringe then the distance b/w them —

$$y_{n+1} - y_n = \frac{D(2n+1)\lambda}{2d} - \frac{D(2n)\lambda}{2d}$$

$$y_{n+1} - y_n = \frac{D\lambda}{d}$$

This is independent of 'n'. So, the distance b/w any 2 consecutive bright or dark fringes is same. This is called fringes width denoted by $\bar{x}$

$$\bar{x} = \frac{D\lambda}{d} \rightarrow (A)$$

Wavelength of Monochromatic light (λ) = $\frac{\bar{x}d}{D}$

*FOLLOWING ARE SOME CONCLUSIONS FROM EQN - A :-

- (i) There is a bright fringe at the centre of the screen.
- (ii) Alternately dark and bright fringe on the both side of the centre
- (iii) Fringe width of dark & bright fringe are same.
- (iv) Fringe width depends upon the wavelength of light.

*ANGULAR FRINGE WIDTH:-

Angular separation b/w consecutive bright or dark fringe is known as Angular fringe width denoted by (ω_0)

$$\omega_0 = \theta_{n+1} - \theta_n$$

$$\theta_{n+1} = \frac{y_{n+1}}{D} - \frac{y_n}{D} = \frac{y_{n+1} - y_n}{D} = \omega_0$$

$$\theta_{n+1} = \frac{D\lambda}{Dd} = \frac{\lambda}{d}$$

$$\boxed{\theta_{n+1} = \frac{\lambda}{d}}$$

REFERENCES:- 12th Book page 49

SUMMARY:-

Constructive Interference $\delta = n\lambda$
 Destructive Interference $\delta = (2n+1)\frac{\lambda}{2}$

fringe width $\bar{x} = \frac{D\lambda}{d}$

TOPIC:- Newton's Ring Experiment

OBJECTIVES:-

OUTCOME:-

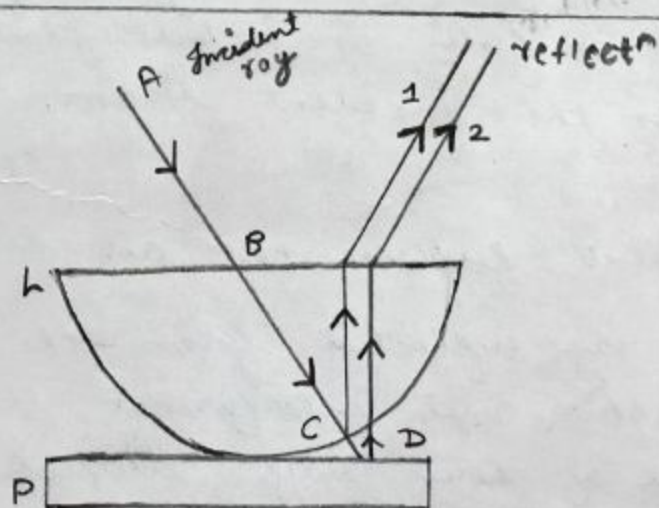
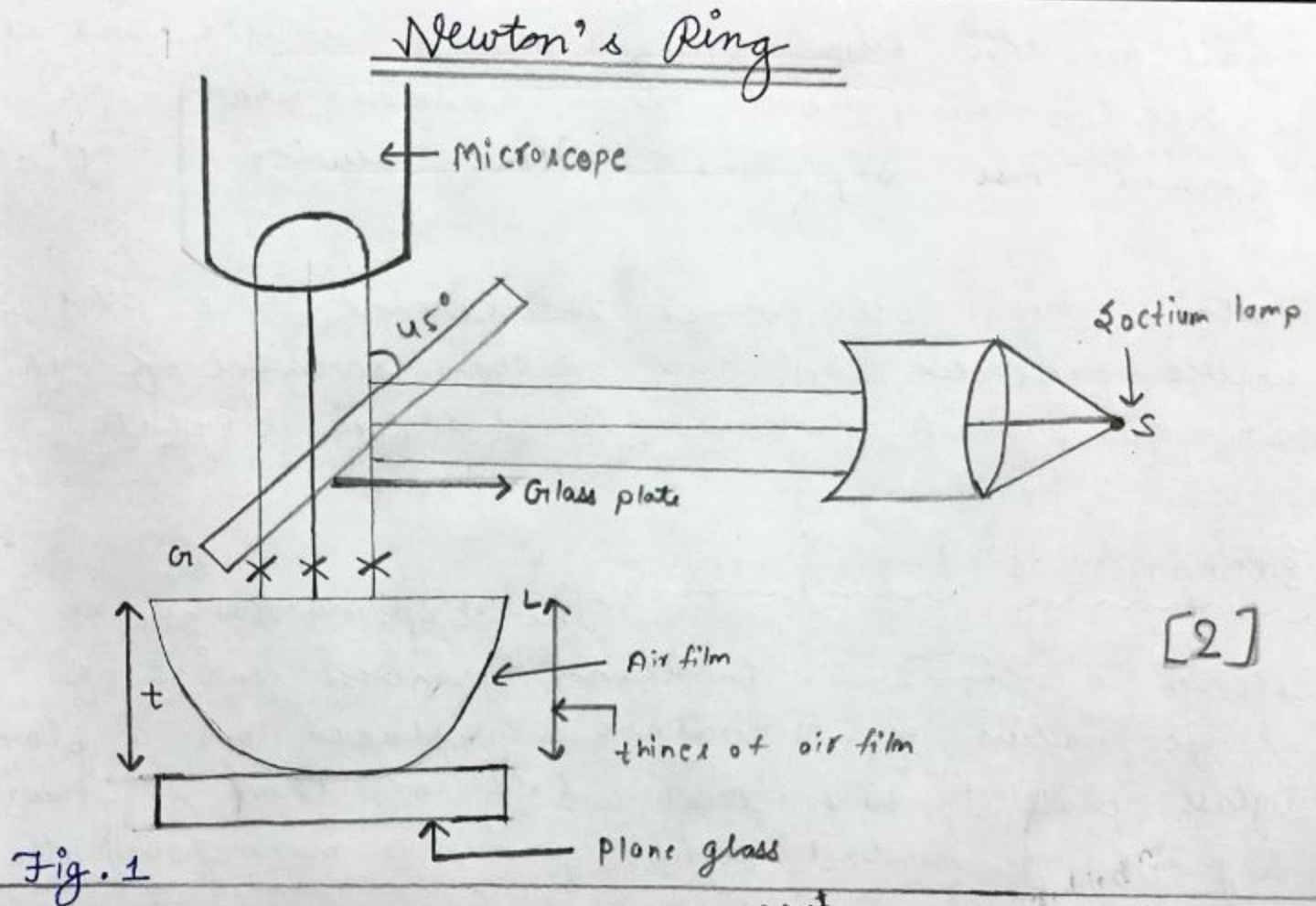


Fig. 2

Formation of Newton's Ring -

When a Plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate. An air film of gradually ^{increasing} thickness from the point of contact is formed b/w the upper surface of the plate and lower surface of the lens. If monochromatic light is fall on this film then alternate bright and dark concentric ring with their centre dark are formed. These rings are called 'Newton's Ring'.

Newton's ring are formed interference b/w the waves reflected Top and bottom surface of the air film, which formed b/w the lens & plate.

Experimental Arrangement -

The Exp. Arrange. as shown in fig. 1, 2. A Plano-convex lens L of large radius of curvature is placed on a plane glass plate P such that both of them is having a point of contact. Light from a monochromatic source is fall on Glass plate G inclined at angle 45° to the incident beam.

The light reflected from Glass plate fall on the air film enclosed b/w L & P . Light ray reflected from the air film superimposed each other and interference takes place. Due to interference of these rays alternate Bright & Dark

Concentric rings are seen in micro-scope as shown in figure. [2]

Thickness of air, maxima (For Bright ring) and minima (For dark ring) and diameter for dark and bright

The rings are observed in reflecting system (reflected light). effective path difference b/w interfering ring is $\Delta = 2\mu t \cos r + \lambda/2$

where, μ = refractive index of air film

t = thickness of air film

r = radius of newton's ring

$\lambda/2$ = Phase change of π on reflection at lower surface of air film. [1]

* First Case -

$$\mu = 1$$

$$r = 0$$

then path difference,

$$\Delta = 2\mu t \cos r + \lambda/2$$

$$\Delta = 2t + \lambda/2 \quad \text{--- (1)}$$

But at the point (p)

$$t = 0$$

$$\Delta = \lambda/2$$

This is the condition for maximum intensity. Hence, The centre of newton's ring is dark.

But, sometimes it is observed that centre is bright. This is because a dust particle comes in b/w the two surfaces at the point of contact.

* Maximum Intensity [Bright fringe]

Path difference $= n\lambda$

$$\Delta = n\lambda$$

From eqⁿ(1)

$$2t + \lambda/2 = n\lambda$$

$$2t = n\lambda - \lambda/2$$

$$2t = \lambda(n - 1/2)$$

$$2t = \lambda/2 (2n - 1) \quad \text{--- (2)}$$

* Minimum Intensity (Dark fringe)

Path difference, $\Delta = (2n + 1)\lambda/2$

$$2t + \lambda/2 = 2 \frac{n\lambda}{2} + \lambda/2$$

$$2t = n\lambda \quad \text{--- (3)}$$

where $n = 1, 2, 3, \dots$

In air film t constant along a circle with its centre at the point contact the fringes are in the form of concentric circle. Since, each fringe is a locus of constant thickness of air film these are known as fringe of constant thickness.

* Diameter for bright ring -

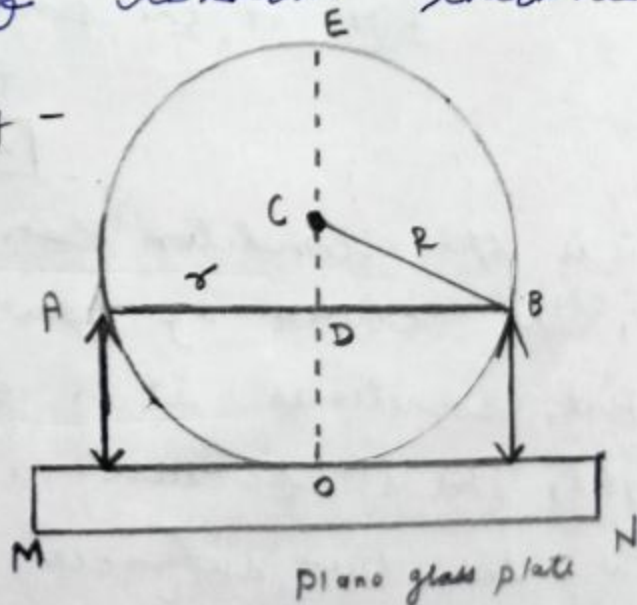
From figure,

A B C D = lens

R = radius of curvature

r = radius of concentric of newton's ring

C = centre of curvature



According to figure,

$$AD \times DB = OD \times DE$$

$\therefore$ (if $C:D$)

$$\Rightarrow AD \times DB = OD \times [DE - OD]$$

$$\Rightarrow r \times r = t (2R - t)$$

$$\Rightarrow r^2 = 2tR - t^2$$

$$\Rightarrow r^2 = 2tR$$

$$\Rightarrow \boxed{2t = \frac{r^2}{R}} \quad \text{--- (4)}$$

If t is very small compare to R so the term t^2 is neglected:-

$$\boxed{r^2 = 2tR}$$

★ For Bright Fringe -

From eqⁿ (2) & (4)

$$\frac{r^2}{R} = (2n-1) \lambda / 2$$

$$\frac{r^2 n}{R} = (2n-1) \lambda / 2$$

$$r_n^2 = R (2n-1) \lambda / 2$$

★ Radius of n th bright ring -

$$r_n = \sqrt{R (2n-1) \lambda / 2}$$

★ Diameter of n th bright ring -

$$D_n = 2r_n$$

$$= 2 \sqrt{R (2n-1) \lambda / 2}$$

$$= \sqrt{4R (2n-1) \lambda / 2}$$

$$D_n \propto \sqrt{2n-1}$$

Thus, the Diameter of bright fringe are proportional to the odd natural number. So, the diameter

of first five bright ring are in ratio
 $D_n \propto \sqrt{2n-1}$

Put $n = 1, 2, 3$

$$\sqrt{1} : \sqrt{3} : \sqrt{5} : \sqrt{7}$$

Separation b/w successive rings

$$D_2 - D_1 = 0.732$$

$$D_3 - D_2 = 0.504$$

$$D_4 - D_3 = 0.410$$

Separation b/w consecutive bright ring rings decreases as the order increases.

Determination of Wavelength of Sodium Lamp & refractive index of liquid.

=> Diameter of the n th ^{dark} ring -

$$D_n = \sqrt{4n \times R\lambda}$$

$$D_n^2 = 4nR\lambda$$

For $(n+p)$ th ^{dark} ring

$$D_{n+p}^2 = 4(n+p)R\lambda$$

$$D_{n+p}^2 - D_n^2 = 4(p)R\lambda$$

$$D_{n+p}^2 - D_n^2 = 4pR\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

=> Diameter of n^{th} bright ring -

$$D_n^2 = 2(2n-1)R\lambda$$

$$D_{n+p}^2 = 2[2(n+p)-1]R\lambda$$

then,

$$D_{n+p}^2 - D_n^2 = 2[2n+2p - \cancel{1} - 2n + \cancel{1}]R\lambda$$
$$= 2[2p]R\lambda$$

$$\boxed{(D_{n+p}^2 - D_n^2)_{\text{air}} = 4pR\lambda} \quad \text{- For Air medium}$$

$$\boxed{(D_{n+p}^2 - D_n^2)_{\text{liq.}} = \frac{4pR\lambda}{\mu}} \quad \text{- For liquid medium}$$

And,

$$(D_{n+p}^2 - D_n^2)_{\text{liq.}} = \left(\frac{D_{n+p}^2 - D_n^2}{\mu} \right)_{\text{air}}$$

$$\boxed{\mu = \frac{(D_{n+p}^2 - D_n^2)_{\text{liq.}}}{D_{n+p}^2 - D_n^2}} \quad \text{- refractive index}$$

* Diameter of n^{th} Dark ring -

$$2t = n\lambda$$

$$t = r^2 R$$

Radius of n^{th} dark ring

$$\frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = nR\lambda$$

Diameter of n^{th} dark ring

$$D_n = 2r_n$$

$$D_n = 2\sqrt{2R\lambda n}, \quad \boxed{D_n = \sqrt{4nR\lambda}} \quad \text{-}$$

Diameter of n^{th} dark

Thus, the diameter of dark ring is $\propto$ to square root of natural no. $\therefore$ $\propto \sqrt{n}$,

$$D_n = \sqrt{4R\lambda} \cdot \sqrt{n} \Rightarrow \boxed{D_n \propto \sqrt{n}}$$

Diameter of first few rings are in ratio

$$1 : \sqrt{2} : \sqrt{3} : \sqrt{4}$$

$$1 : 1.414 : 1.732 : 2$$

Separation b/w two successive rings

$$0.414 : 0.312 : 0.268 \dots \& \text{ on.}$$

Thus, the separation b/w consecutive dark rings also decreases on the order increases.

REFERENCES:-

SUMMARY:-

TOPIC :- Michelson's Machy Interferometer

OBJECTIVES:-

OUTCOME:-

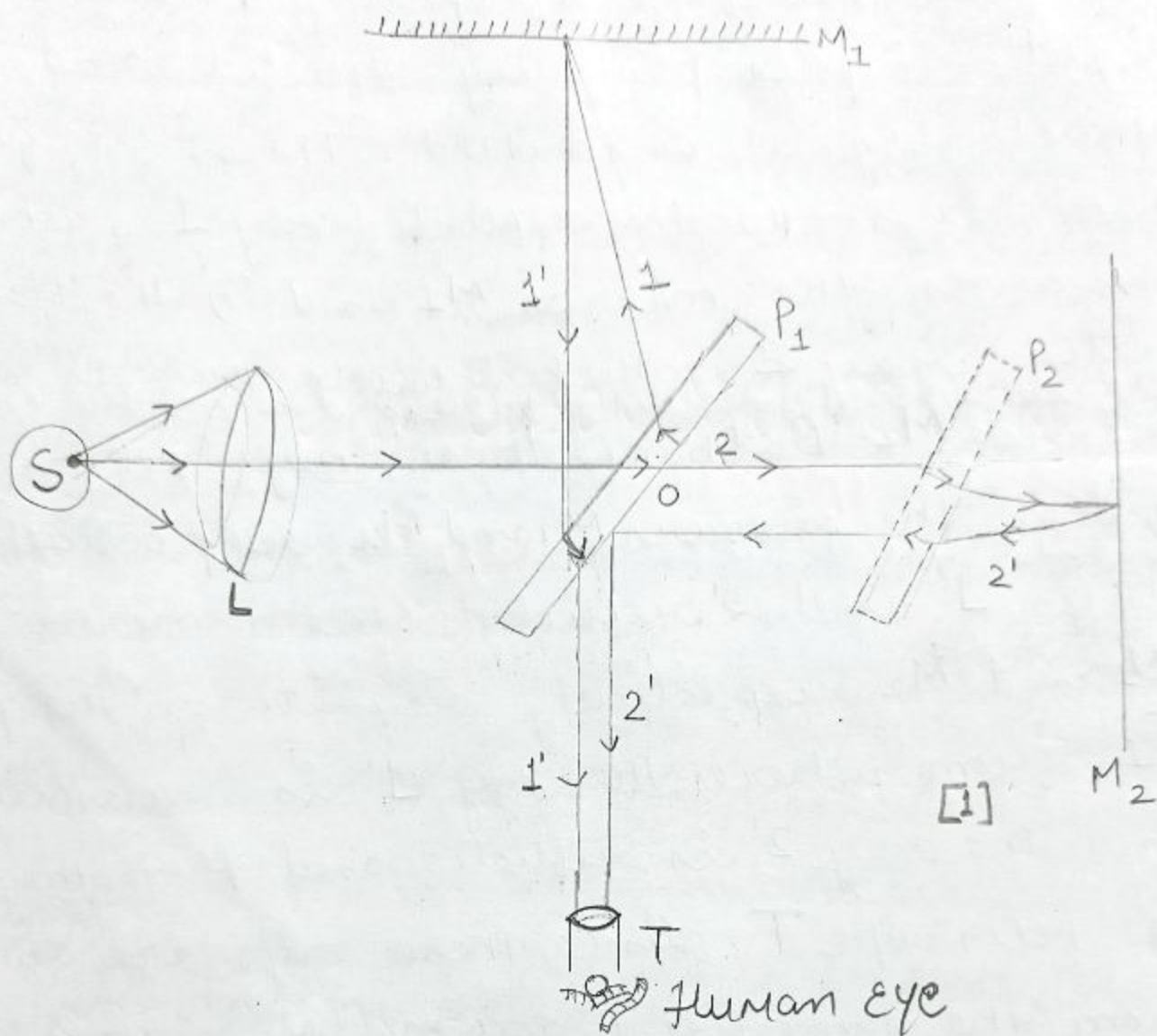
MICHELSON'S INTERFEROMETER

This is an optical instrument by which we can obtain the interference fringes of different shape and by noting their positions the wavelength of light can be measured to a great accuracy.

CONSTRUCTION:-

The optical arrangement of the Michelson's interferometer. It consists of two plane mirrors M_1 and M_2 and the optically plane glass plate P_1 and P_2 of the same thickness. The mirrors M_1 and M_2 are silvered on their front surfaces and they are mounted on two arms mutually perpendicular to each other. Each mirror

is provided with the three levelling screws at its back, by which it can be tilted on the horizontal and vertical axis. The mirror M_2 is kept stationary while the mirror M_1 can be moved on a scale by means of an accurate screw such that its new position is exactly parallel to its previous position. The distance moved by an mirror can be read on a scale with an accuracy upto 10^{-5} cm (or 10^{-7} m). plates P_1 and P_2 are kept exactly parallel to each other and inclined at an angle of 45° from the mirrors M_1 and M_2 . The surface of the plate P_1 which is towards the plate P_2 , is partially silvered and the plate P_2 is completely transparent. plate P_2 is known as the Compensating plate. The fringes are seen by a telescope T .



EXPERIMENTAL ARRANGEMENT OF MICHELSON'S INTERFEROMETER

WORKING :- Light rays from an extended monochromatic source S are made incident on the plate P_1 in form of a parallel beam through a convex lens L . At the point O of the partially silvered surface of plate P_1 , the incident light is partially

silvered surface as plate, the incident light ray is partially reflected and partially transmitted. Thus, we get two rays - (i) the reflected ray 1, which travels towards the mirror M_1 and (ii) the transmitted ray 2, which travels towards the mirror M_2 . Both of these rays get reflected from the mirrors M_1 and M_2 respectively. The rays 1' and 2' reflected from mirrors M_1 and M_2 respectively reunite on the plate P_1 from where the ray 1' is transmitted and the ray 2' is reflected and then they enter the telescope T. Since these rays are obtained from the same light source, hence they are coherent. These rays interfere due to which fringes are formed which can be seen through the telescope.

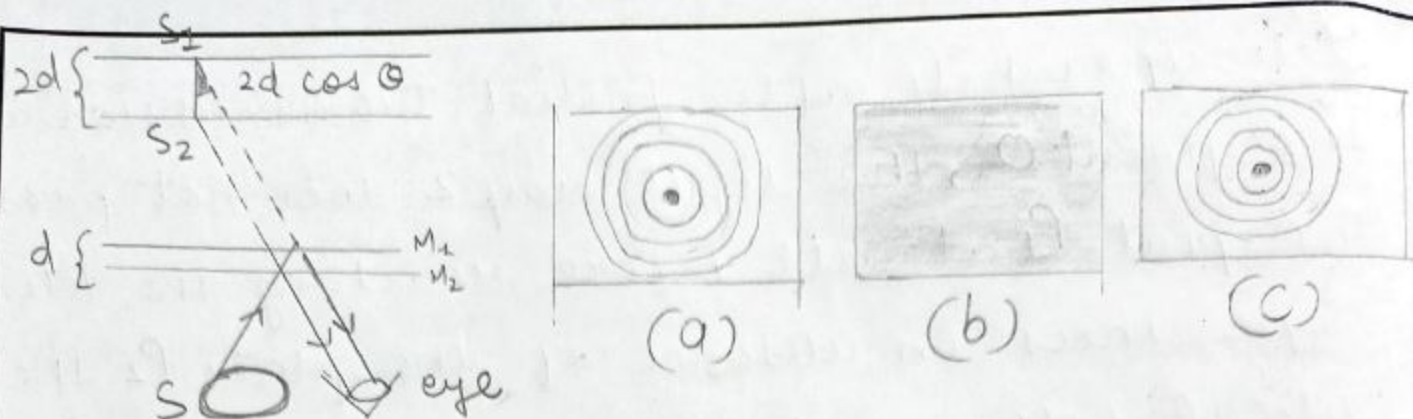
Function of Compensating plate P_2 - we can understand the function of the compensating plate P_2 after partial reflection at the point O the light ray 1 passes the plate P_1 two times before reaching the

telescope while after partial transmission at the point O , the light ray 2 does not pass the plate P_1 at all before reaching the telescope. Thus in absence of the plate P_2 the optical paths of these rays are not equal. To make these paths equal, a plate P_2 of thickness and material same as that of the plate P_1 is used which is kept in the path of the ray 2, exactly parallel to the plate P_1 . Thus, the plate P_2 is called the Compensating plate. Remember that the only difference in the plate P_1 and plate P_2 is that the plate P_2 is perfectly transparent while the plate P_1 is partially silvered.

[2]

PRINCIPLE OF FORMATION OF CIRCULAR FRINGES

Circular fringes are generated with monochromatic light in a Michelson interferometer. Here the mirror M_1 and the virtual mirror M_2' which is the image of M_2 must be parallel



The source is an extended one S_1 and S_2 are the virtual images of type source due to M_1 and M_2' . The d/B S_1 & S_2 is $2d$, when the d/B M_1 & M_2' is d . The path difference b/w the two beams will be $2d \cos \theta$. Hence, the rays for which $2d \cos \theta = n\lambda$ will reinforce to produce maxima. These circular fringes which are caused by interference with a phase difference determined by the inclination θ are all known as fringes of equal inclination or Haidinger's fringes.

REFERENCES:-

SUMMARY:-

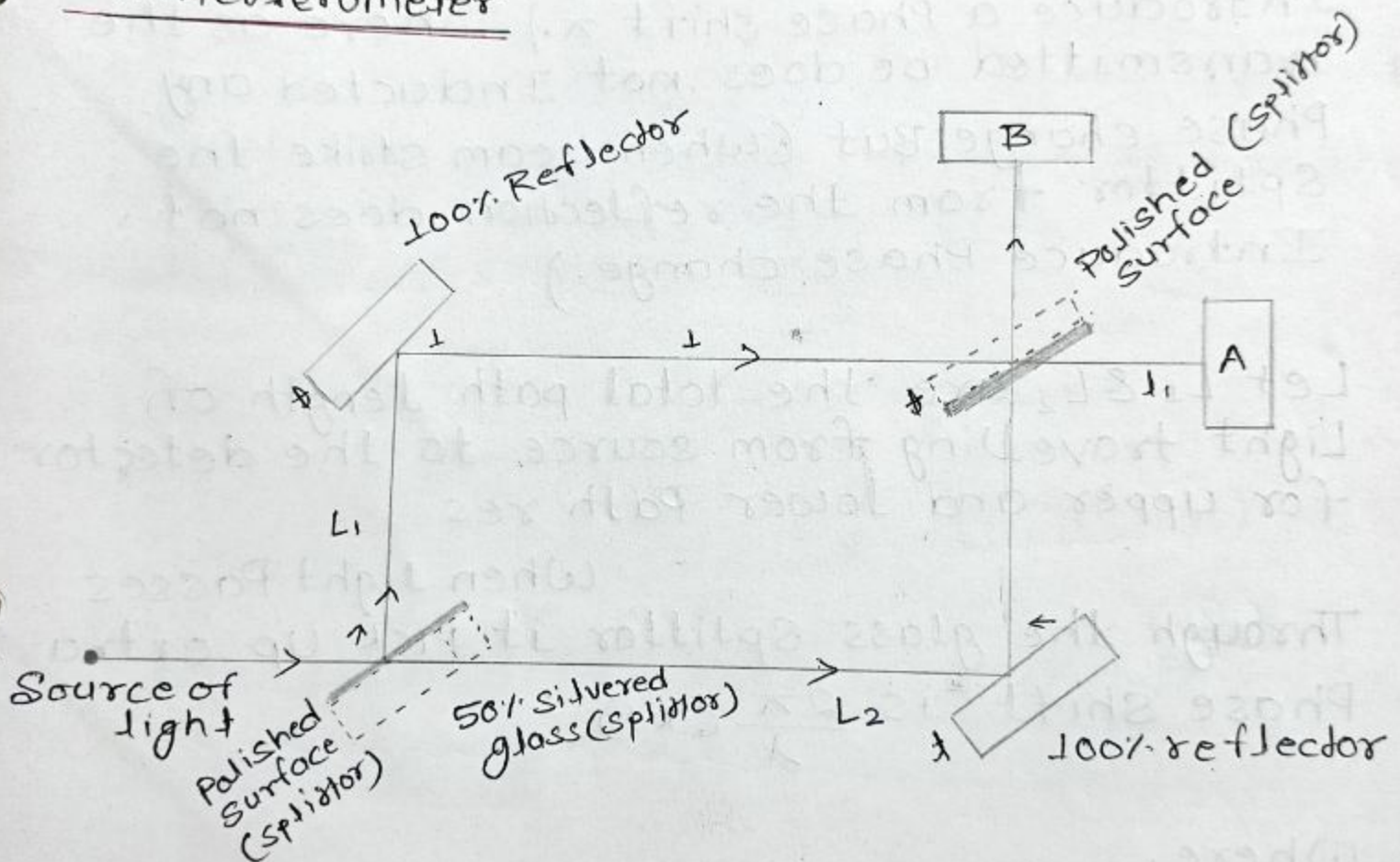
TOPIC :- Mach Zehnder Interferometer

PAGE NO

OBJECTIVES:-

OUTCOME:-

* Mach - Zehnder Interferometer



It is a device for demonstrating interference by division of amplitude. It is used to determine the relative phase change variation b/w two collimated beams by splitting light from a single source.

It is an experiment a light beam is 1st split into two parts by the splitter and then recombine by the second splitter.

Let beam splitter is a piece of glass with coating on a front surface when light strike front surface of a splitter there it divide two rays reflector and transmitting.

Now,

(A Reflection)

Introduce a phase shift π where as the transmitted beam does not induce any phase change but (when beam strike the splitter from the reflection does not introduce phase change.)

Let L_1 & L_2 are the total path length of light travelling from source to the detector for upper and lower path res

When light passes through the glass splitter it pick up extra phase shift is $\frac{2\pi}{\lambda} (kt)$

Where,

t = optical path length of splitter

1. >

Upper path on the way to detector "A" has phase change π at first reflection (silvered glass piece) And again phase

change(π) by the reflector.)

There $2\pi/\lambda L_1 =$ Total Path length and
 $2\pi/\lambda \cdot t =$ Extra phase picked up
in travelling through the glass splitter
Then effective path travelled by the upper
light at detector A is given as,

$$\begin{aligned} & \pi + \pi + \frac{2\pi}{\lambda} L_1 + \frac{2\pi}{\lambda} t \\ & 2\pi + \frac{2\pi}{\lambda} (L_1 + t) \quad \text{--- (i)} \end{aligned}$$

Similarly, effective path travel for the
lower phase path the effective path
difference is

$$\begin{aligned} & \frac{2\pi}{\lambda} t + \frac{2\pi}{\lambda} L_2 + \pi + \pi \\ & \frac{2\pi}{\lambda} (t + L_2) + 2\pi \quad \text{--- (ii)} \end{aligned}$$

by (ii) - (i) we have
the path difference is given by

$$A = \frac{2\pi}{\lambda} (L_1 - L_2)$$

Similarly, Phase difference b/w two
paths on detector B.

$$\begin{aligned} AB &= \left[\left\{ \frac{2\pi}{\lambda} L_1 + \frac{2\pi}{\lambda} 2t + 2\pi \right\} - \left\{ \pi + \frac{2\pi}{\lambda} L_2 + \frac{2\pi}{\lambda} t \right\} \right] \\ &= S_B = \pi + \frac{2\pi}{\lambda} (L_1 - L_2) \\ & \quad \boxed{S_B = D (L_1 - L_2)} \end{aligned}$$

Thus,

$$S_A = D(L_1 - L_2)$$

Where

Constructive Interference on the path to detector (A) and destructive Interference on the path to detector (B.)

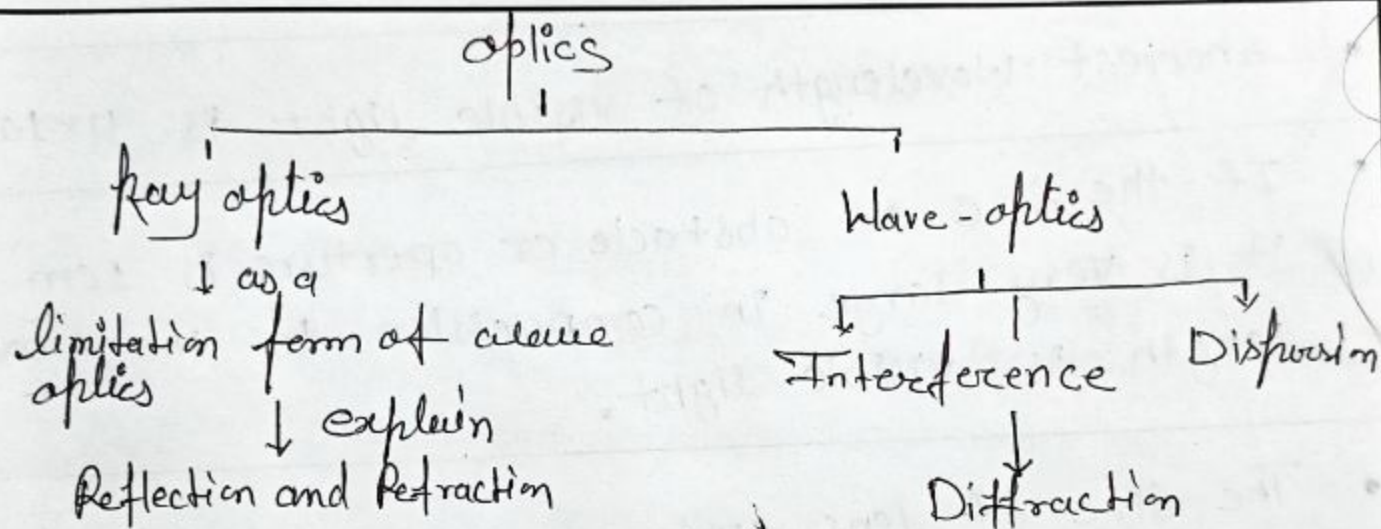
REFERENCES:-

SUMMARY:-

TOPIC:- Diffraction, Rayleighs Criterion

OBJECTIVES:-

OUTCOME:-



• Define Diffraction : →

- Wave nature of light can explain all the optical phenomenon.
- The phenomenon of diffraction of light i.e bending of light at the corner of an obstacle or aperture is possible only if the size of the obstacle or aperture is of the order of wave length of light

- If the size of aperture is very large as compared to the wave-length of light. The effect of diffraction is negligible and light travels nearly in a straight line path then light is in form of Ray.

- Shortest Wavelength of visible light is $4 \times 10^{-7} \text{ m}$

- If the size of obstacle or aperture is 1cm it is very large in comparison to the wave length is visible light.

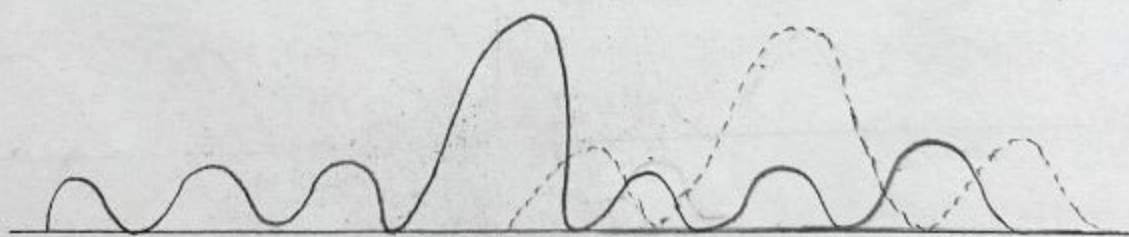
- The size of lens, prism, mirror and optical instruments is very large as compared to the λ of light (visible) then light can be considered as a ray travelling in a straight line. Thus the ray optics is a limiting case of wave optics.

Diffraction :- The bending of wave around an obstacle or Aperature is known as diffraction sound waves Bend around walls and building i.e also diffraction.

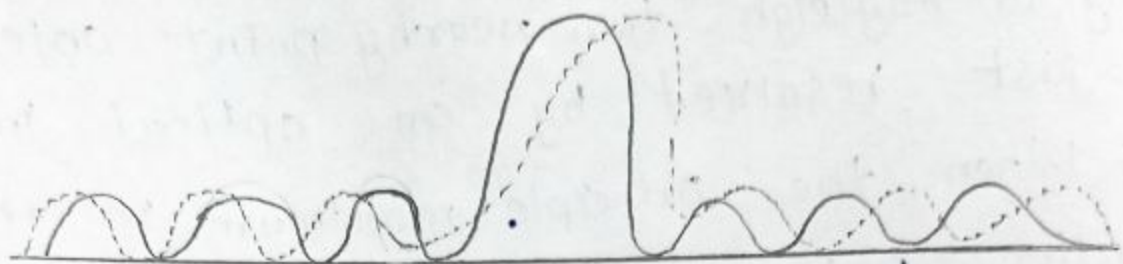
Rayleigh's criterion of Resolution $\rightarrow$

According to Rayleigh "two nearby point objects are just resolved by an optical instrument, when the principle maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa."

The same thing happens in case of spectral lines of two different wave lengths, the lines are resolvable, when the principle maximum due to one wave length falls over the first minimum due to the other and vice versa.



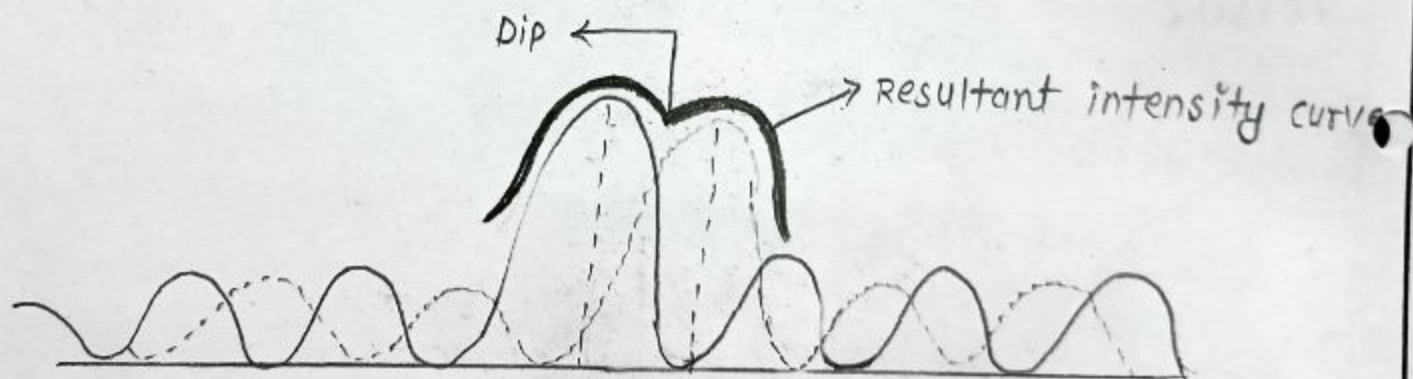
Easily Resolvable



or



Not Resolvable



Just resolvable

TOPIC :- Grating, Resolving power of Grating

OBJECTIVES:-

OUTCOME:-

Resolving Power of an optical Instrument:

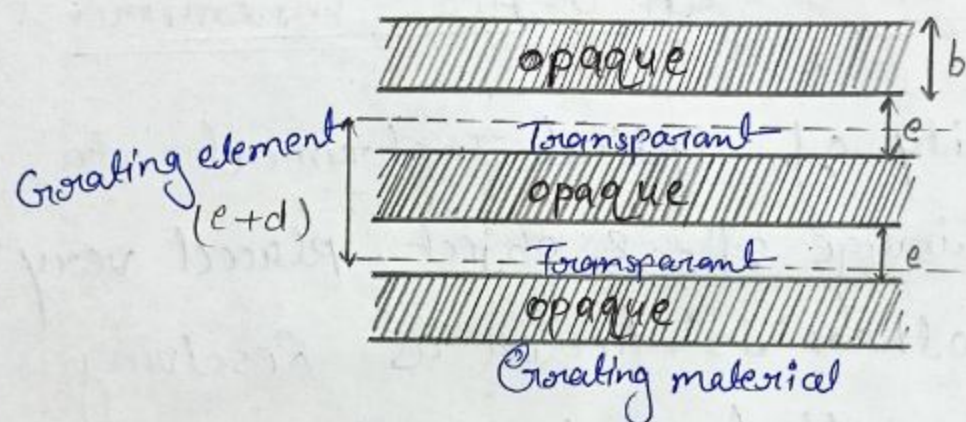
The optical ability of optical Instrument to form separate image of 2 object, placed very close to each other is known as Resolving power of the optical Instrument.

The resolving power of an instrument is inversely proportional to its resolving limit -

$$\text{Resolving power} = \frac{1}{\text{Resolving limit}}$$

Grating :-

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating.



Resolving power of Grating :-

It is defined as its ability to form separate diffraction maxima of 2 wavelength which are very closed to each other.

If $\Delta\lambda$ is the smallest difference in 2 wavelength which can just resolved by a Grating and λ is the wavelength of either of them then

$\frac{\lambda}{\Delta\lambda}$ is known as Resolving power of the Grating.

i.e. The resolving power of the diffraction grating represent its ability to form separate lines for wavelength very close together.

Derivation: —

Consider P_1 is position of the n th principle maximum of spectral line of wavelength λ and P_2 is position of the n th principle maximum of spectral line of wavelength $(\lambda + d\lambda)$

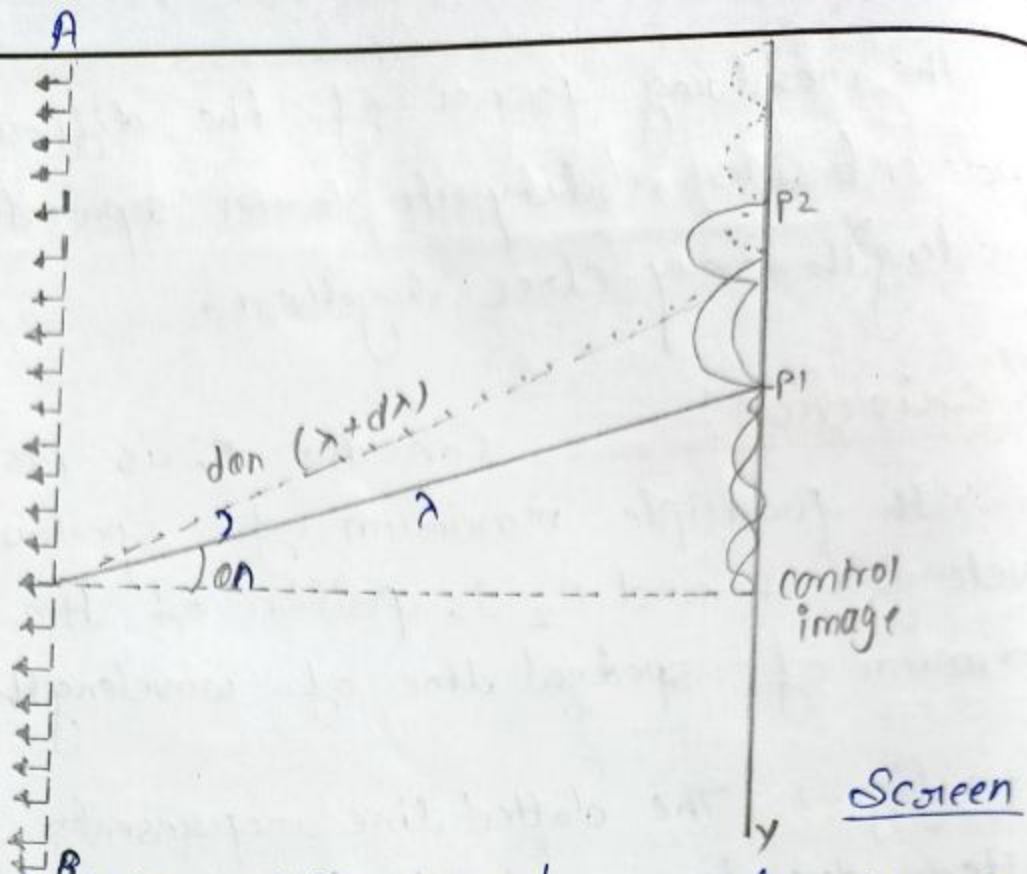
from fig $\Rightarrow$ The dotted line represents the diffraction pattern due to wavelength $(\lambda + d\lambda)$ and solid line show the diffraction pattern "due to wavelength λ ."

According to Rayleigh's Condition of resolution

The two wavelength can be resolved, if the position of P_2 coincides with the first minimum due to wavelength " λ ".

The principle maximum of wavelength λ in a direction θ is

$$(e+d) \sin \theta = n \lambda \quad \text{--- ①}$$



and the principle minimum of wavelength λ in a direction θ_n is

$$N(e+d)\sin\theta_n = m\lambda \quad \text{--- (1)}$$

Where m is all the integral values except $0, N, 2N, \dots, nN$ because -

For these values of m the condition for maximum is satisfied. N is total number of slits on the grating.

Now in the direction $(\theta_n + \delta\theta)$

Therefore, the first minimum adjacent to n th principle maximum in the direction $(\theta_n + \delta\theta)$

Putting the value of -

$$m = nN + 1$$

$$\theta_n = \theta_n + d\theta_n \quad \text{in equation - (2)}$$

$$N(e+d) \sin(\theta_n + d\theta_n) = n(N+1)\lambda \quad \text{--- (3)}$$

Similarly for wavelength $(\lambda + d\lambda)$

The principle maximum in the direction $(\theta_n + d\theta_n)$

From equation - (1) putting $\theta_n = \theta_n + d\theta_n$
 $\lambda = \lambda + d\lambda$

$$(e+d) \sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \text{--- (4)}$$

multiplying by N on above equation both side

$$\Rightarrow N(e+d) \sin(\theta_n + d\theta_n) = nN(\lambda + d\lambda) \quad \text{--- (5)}$$

From equation (3) and (5) equating

$$\Rightarrow (nN+1)\lambda = nN(\lambda + d\lambda)$$

$$\Rightarrow \cancel{nN}\lambda + \lambda = \cancel{nN}\lambda + nNd\lambda$$

$$\Rightarrow \lambda = nNd\lambda$$

$$\Rightarrow \boxed{\frac{\lambda}{d\lambda} = nN} \quad \text{--- (6)}$$

This is resolving power of diffraction Grating which is directly proportional to

- 1) The order of the spectrum
- 2) Total no. of line on grating -

From equation - (1)

$$n = \frac{(e+d) \sin \theta_n}{\lambda} \quad \text{--- (2)}$$

From equation (6) and (7)

$$\boxed{\frac{\lambda}{d\lambda} = \frac{N(e+d) \sin \theta_n}{\lambda}}$$

where the terms $N(e+d)$ is called total width of the grating.

REFERENCES:-

SUMMARY:-

TOPIC: - Fraunhofer Diffraction by the Single Slit.

OBJECTIVES:-

OUTCOME:-

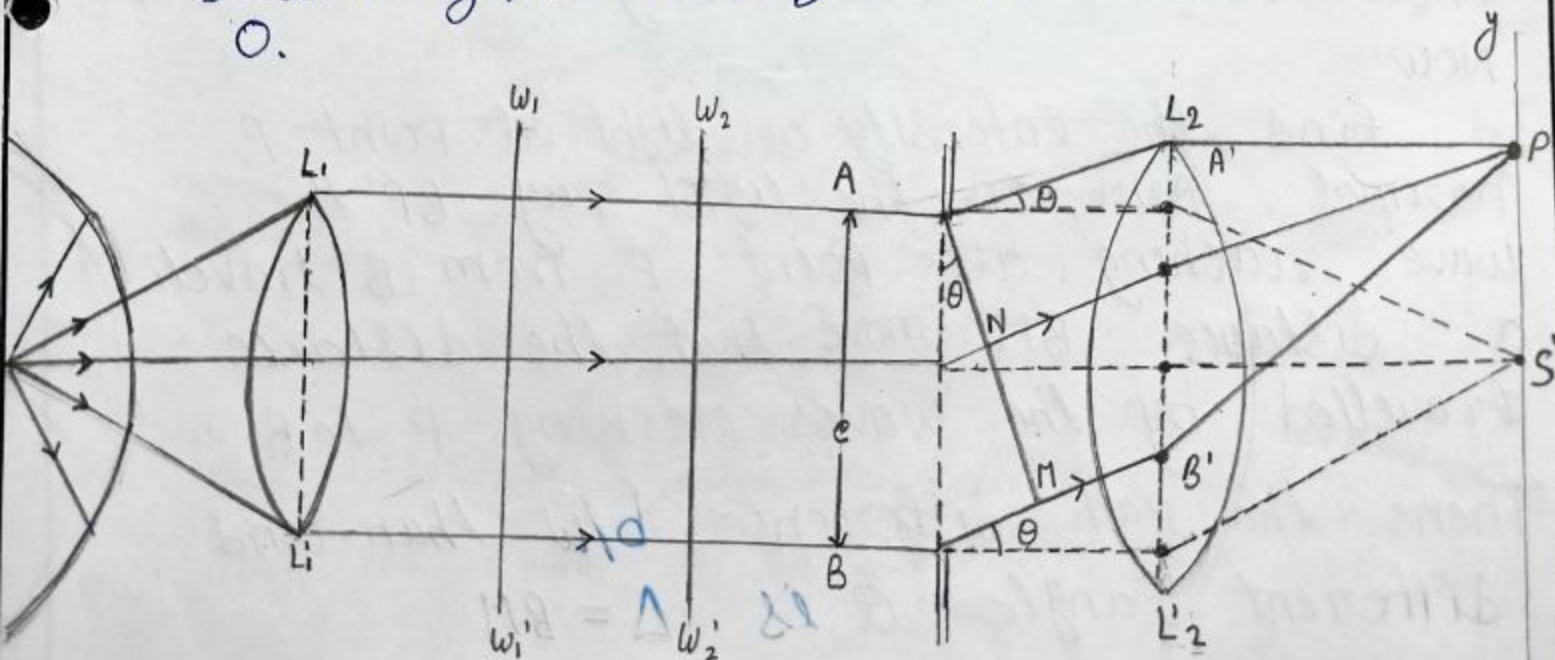
FRESNAL DIFFERACTATION BY THE SINGLE SLIT

The light beam from a monochromatic point source S is converted into a beam of light by a convergent lens (L_1 & L_2)

Now

This beam is incident normally on a slit AB of width c .

- The incident light wavefront w, w' and w_2, w_2'
- The intensity of light at the successive maxima decreases by the intensity of light at each minima is 0.



EXPLANATION

A/c to the Huygen's wave theory the plane wave front incident normally on the slit AB. and the position of the incident wavefront at any instant t is AB from each point of wavefront AB secondary wavelets are emitted out in each direction.

The undeviated wave fronts present in the direction of incident light get focused at point S. on the screen while the wavelets diffracted at an angle θ , are focused at point P on the screen.

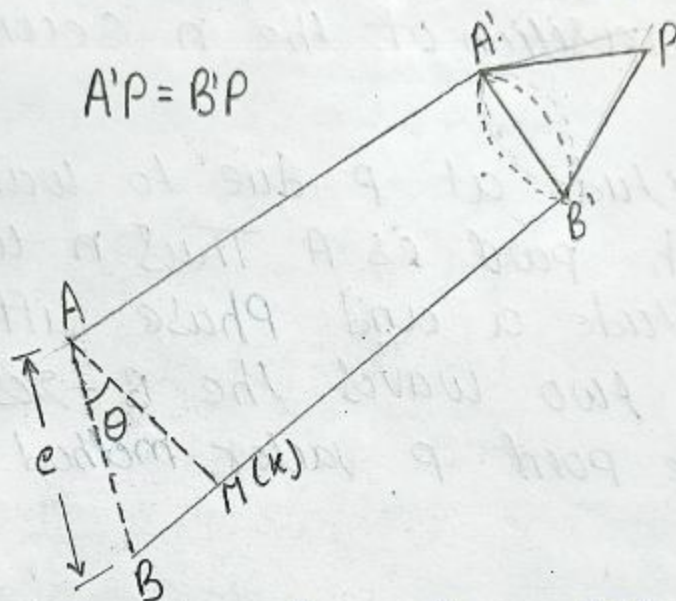
Science the optical path of the point S from each point of the plane AB is equal hence all the secondary wavelets reach the point S in same phase. This there is maximum intensity of Now

Find the intensity of light at point P normal, ANM on the light ray BP the wave reaching at point P from B travels a distance BM more than the distance travelled by the waves reaching P to B

There the path difference b/w them and different angle θ is $\Delta = BM$

ANALYSIS

The width of slit is very small in comparison to the aperture of L_2 ray from A and B falls only on a very small portion of lens L_2 and therefore the lens may be considered perpendicular to the diffracted ray as shown in fig.



AK is perpendicular from A on BB' difference of the rays meeting at P

$$\Delta AK^m$$

$$\sin \theta = \frac{BK^m}{AB} = \frac{BM}{AB}$$

$$\Delta = BK^m = AB \sin \theta, \quad BM = AB \sin \theta$$

$$= e \sin \theta$$

$$= e \sin \theta$$

$$\Delta = BM = BK^m = e \sin \theta$$

Phase difference increases from $\theta = \frac{2\pi}{\lambda}$

$$\begin{aligned} \text{Therefore Phase difference} &= \frac{2\pi}{\lambda} \text{ Path difference} \\ &= \frac{2\pi}{\lambda} (e \sin \theta) \quad \dots (1) \end{aligned}$$

If slit assumed made up of equal path let amplitude of each secondary wave is 'a'

Then phase difference b/w any two successive waves

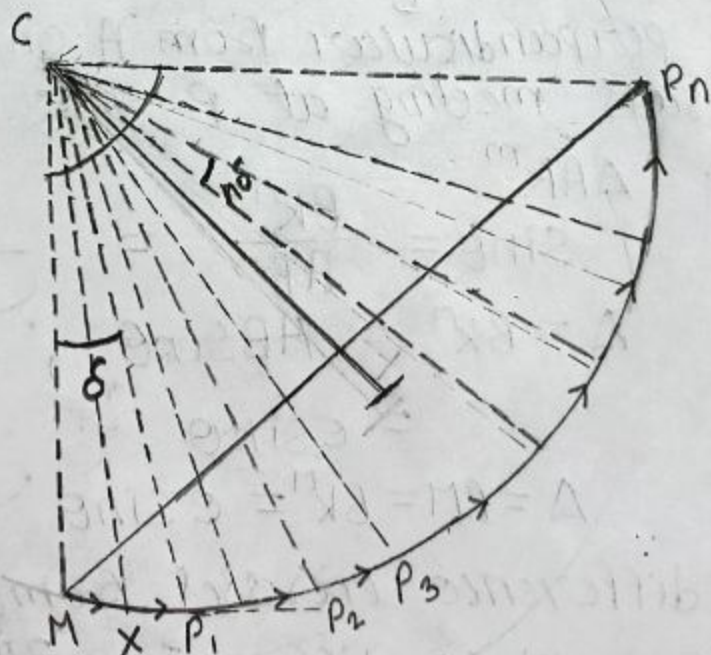
$$= \frac{1}{n} \left(\frac{2\pi}{\lambda} \right) e \sin \theta$$

$$= \frac{1}{n} \left(\frac{2\pi}{\lambda} \right) e \sin \theta \quad \text{--- (2)}$$

Thus the resultant in a direction θ due to the superposition of the n secondary waves

Let amplitude at P due to wave obtained from each part is A Thus n waves each of amplitude a and phase difference

b/w two waves the resultant amplitude at the point P vector method



draw vector $MP, P_1P_2, P_2P_3, P_3P_4 \dots$
 angle b/w the vector is δ The vector MP_n
 Joining first vector M . Let the magnitude of
 resultant Vector MP_n is R_0 .

Let CX normal drawn from C on MP and
 CY on MP_n from C

$$\text{from } \triangle CXM \cdot \frac{MX}{MC} = \sin \frac{\delta}{2}$$

$$MX = MC \sin \frac{\delta}{2}$$

$$\text{But } MX = \frac{1}{2} MP_1 = \frac{A}{2}$$

$$\frac{A}{2} = MC \sin \frac{\delta}{2} \dots \dots (iii)$$

similarly

$$\triangle CYM = \frac{MY}{MC} = \sin \left(\frac{n\delta}{2} \right)$$

$$MY = MC \sin \left(\frac{n\delta}{2} \right)$$

$$MY = \frac{1}{2} MP_n = \frac{1}{2} R_0$$

$$\frac{R_0}{2} = MC \sin \left(\frac{n\delta}{2} \right) \dots \dots (iv)$$

from eqn (iii) and (iv)

$$\frac{R_0/2}{A/2} = \frac{MC \sin(n\delta/2)}{MC \sin(\delta/2)}$$

$$R_0 = A \frac{\sin(n\delta/2)}{\sin(\delta/2)}$$

$$R_0 = A \times \frac{\sin(n \cdot \frac{2\pi}{n\lambda} \cdot e \sin \theta)}{\sin(\frac{2\pi}{n\lambda} e \sin \theta/2)}$$

$$R_0 = A \times \frac{\sin(\pi d \sin \theta)}{\sin(\pi/n d \sin \theta)}$$

$$R_0 = \frac{n e \sin \theta}{1} = d \sin \theta$$

$$R_0 = \frac{A \sin d}{\sin(\frac{d}{n})} = \frac{A \sin d}{d/n}$$

Since n is very large so d/n is very small then
 $\sin d/n = d/n$

$$R_0 = nA(\sin \alpha/2) \dots \dots \textcircled{v}$$

If all the waves are in same phase Then

$$\lim_{\alpha \rightarrow 0} \left(\frac{\sin \alpha}{\alpha} \right) = 1$$

$$R_0 = nA$$

from eqn (v)

$$R = R_0 \left(\frac{\sin \alpha}{\alpha} \right)$$

This is the resultant amplitude at diffraction angle θ .
Hence. The resultant intensity at $[P, \theta]$

$$I \propto R^2$$

$$I = kR^2$$

$$I = kR_0^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$I_0 = kR_0^2$$

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

condition for maxima and minima

$$\frac{dI}{d\alpha} \left(\frac{\sin^2 \alpha}{\alpha^2} \right) = 0$$

$$A^2 \left(\frac{2 \sin \alpha}{\alpha} \right) \left[\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} \right] = 0$$

Case-I $\rightarrow$ When $\frac{\sin \alpha}{\alpha} = 0$ or $\sin \alpha = 0$

$$\alpha = \pm n\pi \quad \{\alpha \neq 0\} = \frac{2\pi \sin \theta}{\lambda} = n\pi$$

$$\sin \theta = \pm n\lambda$$

$\therefore I=0$ at $\sin \alpha/2 = 0$
direction first minima from normal

$$\sin \theta_1 = \pm \lambda/e$$

$$\sin \theta_n = \pm n\lambda/e$$

Case-II $\rightarrow$

Second bracketed term direction of the maxima

$$\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

REFERENCES:-

$$A \sin \alpha = \alpha$$

SUMMARY:-

TOPIC :- Semiconductor,

OBJECTIVES:-

OUTCOME:-

Semiconductor

Definition:- A Semiconductor is a material which has an electrical conductivity value falling between that of a conductor, such as copper and an insulator, such as glass. Its resistivity falls as its temperature rises; metals behave in the opposite way.

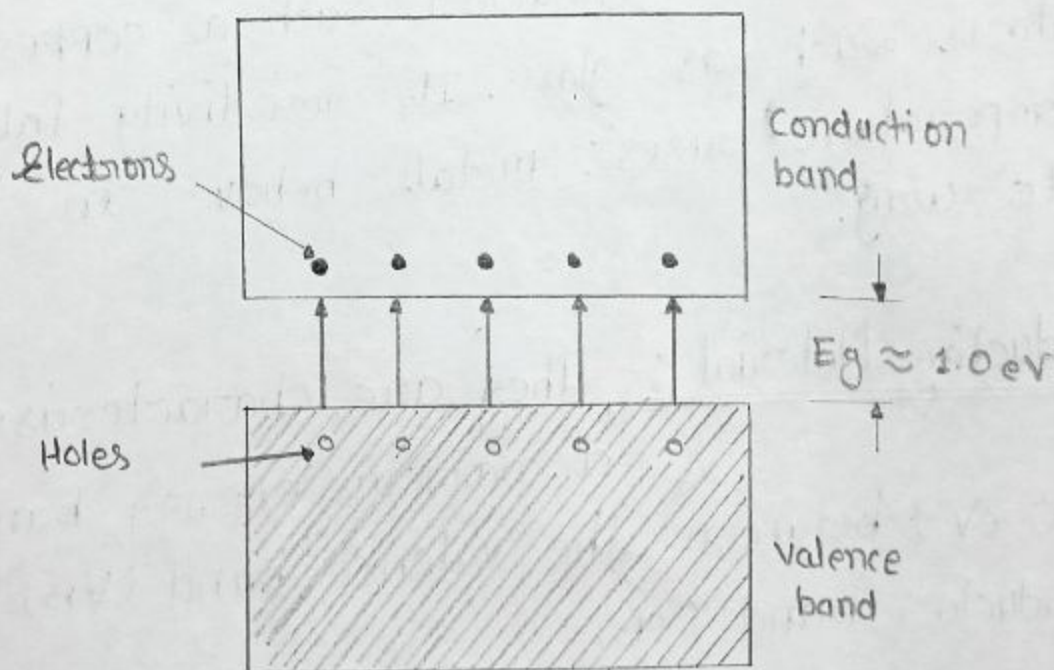
Semiconducting Material :- They are characterised by a narrow energy band gap ($E_g \sim 1.0 \text{ eV}$) between the valence band (VB) and the conduction band (CB)

Element of group IV elements like germanium (Ge) and Silicon (Si) are typical semiconductor materials

having Energy Band gap of 0.72 eV and 1.10 eV respectively. Other group IV element like diamond exhibits insulator properties even at very high temperature because its bond strength is very high.

Group IV : $\underbrace{\text{C (Diamond)}}_{\text{insulator}} > \underbrace{\text{Si > Ge}}_{\text{Semiconductor}} > \underbrace{\text{Sn > Pb}}_{\text{metallic conductor}}$ (Bond Strength)

An applied electric field cannot give so much energy to the valence electrons that they could cross the Energy gap E_g and enter the CB. Hence Pure Semiconductors are insulators at low temperatures.



Energy Band Diagram of a Semi-conductor Material

Semiconductors are classified as:

- i) Element and compound Semiconductors, or
- ii) Intrinsic and extrinsic Semiconductors.

Element Semiconductors: These are basically group IV element of the periodic table

C, Si, Ge, Sn, Pb. They are having diamond cubic structure i.e., tetrahedral coordination. Bonding is covalent due to hybridization.

	C	Si	Ge	Sn (grey)	Pb
Band gap (E_g)	5.3 eV	1.1 eV	0.72 eV	0.08 eV	-

Compound Semiconductors: As conductivity and band gap are limited for elemental Semiconductors hence they are not much useful.

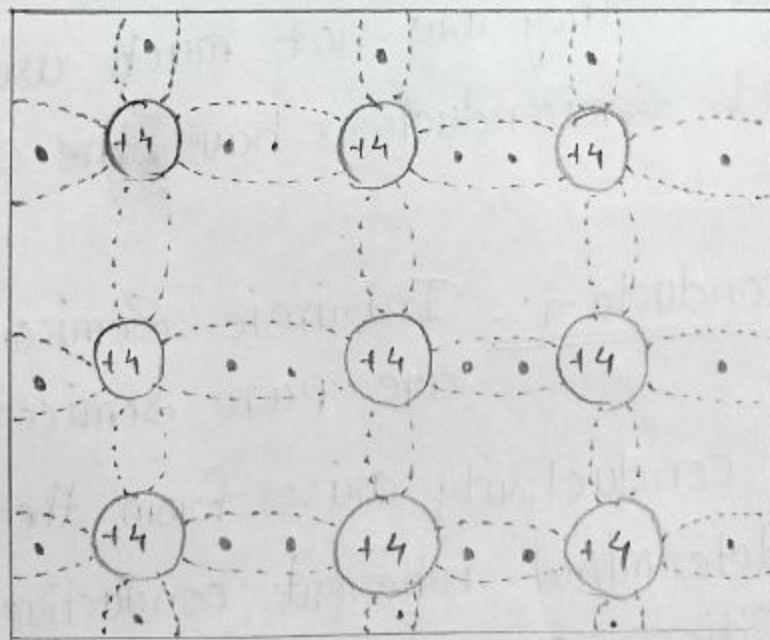
All these compounds Semiconductors have the average charge of four.

Intrinsic Semiconductor: Intrinsic Semiconductors are pure Semiconductors

whose electrical conductivity arises from thermal excitation and determined inherent conductive properties. Pure elemental Silicon and Germanium are intrinsic

Semiconducting materials.

Let us consider an intrinsic semiconductor like Ge and Si which are both tetravalent. Ge and Si have four valence electrons each of which is shared by one of its four nearest neighbouring atoms. Due to this sharing covalent bonds are formed, which are shown by dashed lines. At temperature near absolute zero, all valence electrons are tightly bounded and so no free electrons are available to conduct electricity.



Bonding in an intrinsic Semiconductor

Extrinsic Semiconductor: As we know that in intrinsic semiconductor

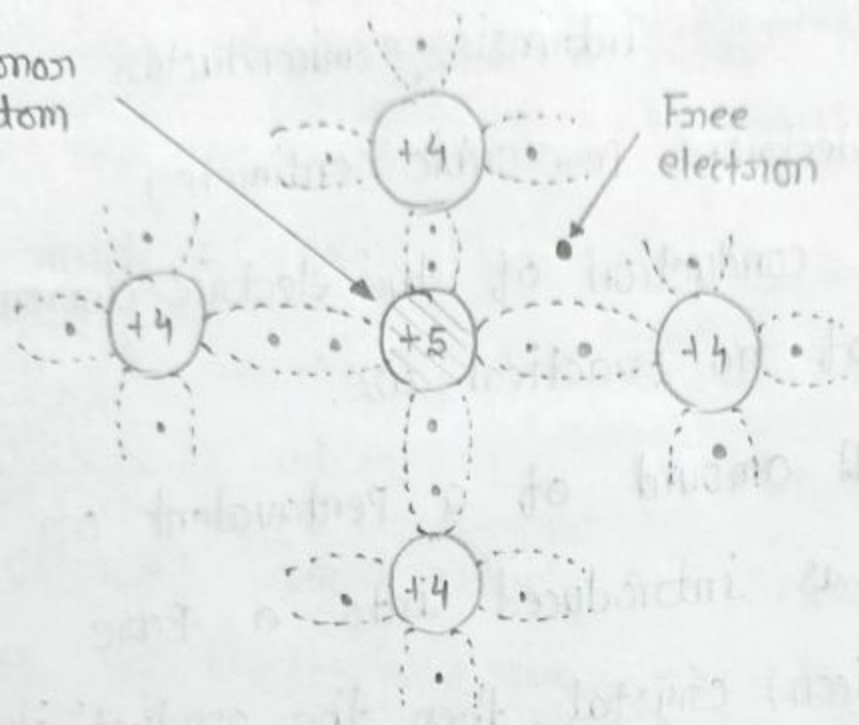
only about 10^9 electrons per cubic centimeter contribute to the conduction of the electric current, hence these are of no practical use.

If a small amount of a pentavalent or trivalent impurity is introduced into a pure Germanium (or Silicon) crystal, then the conductivity of the crystal increases appreciably and the crystal becomes an 'extrinsic' Semiconductor. The process of adding small amounts of impurities to the semiconductor material is known as doping. The impurity element that replaces a regular lattice atom is known as dopants.

When a small amount of a pentavalent atom is added to Ge (or Si) four of these valence electrons form regular electrons pair bonds with their neighbouring Ge (or Si) atoms. The fifth electron however is loosely bounded to Si,

Donor atom

Free electron



REFERENCES:-

SUMMARY:-

TOPIC :- P-N-Junction

OBJECTIVES:-

OUTCOME:-

Junction
P-N DIODE

A diode is a two terminal device which acts as a switch since it offers low resistance to the flow of current in the forward biased condition and a high resistance in the reverse biased condition

In this diode current due to minority charge carriers is neglected, the current through P-N Junction flow only from $P \rightarrow N$ and not from $N \rightarrow P$

In P-N Junction current flows through the Junction only in one direction $P \rightarrow N$

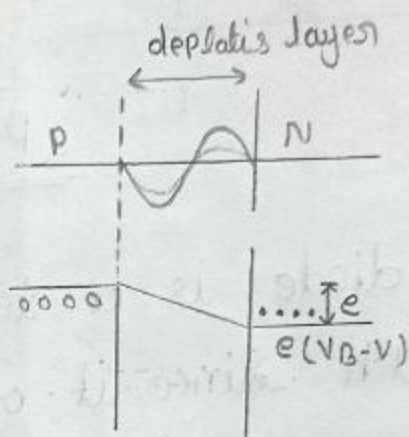
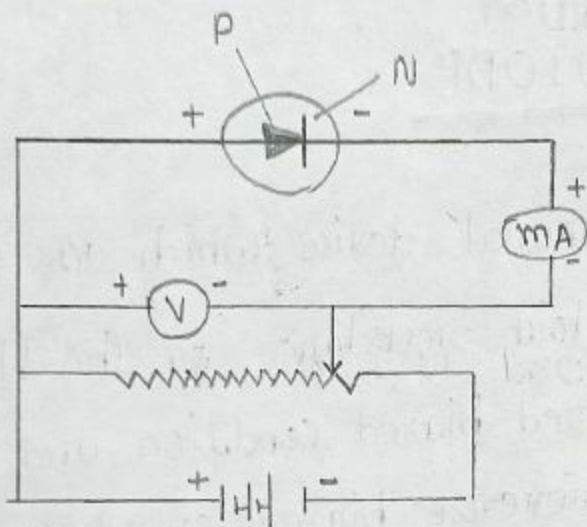
Hence this Junction behaves like a Vacuum diode valve
SYMBOL OF P-N DIODE -



P is +ve terminal (Majority Hole)
 N is -ve terminal (Majority e^-)

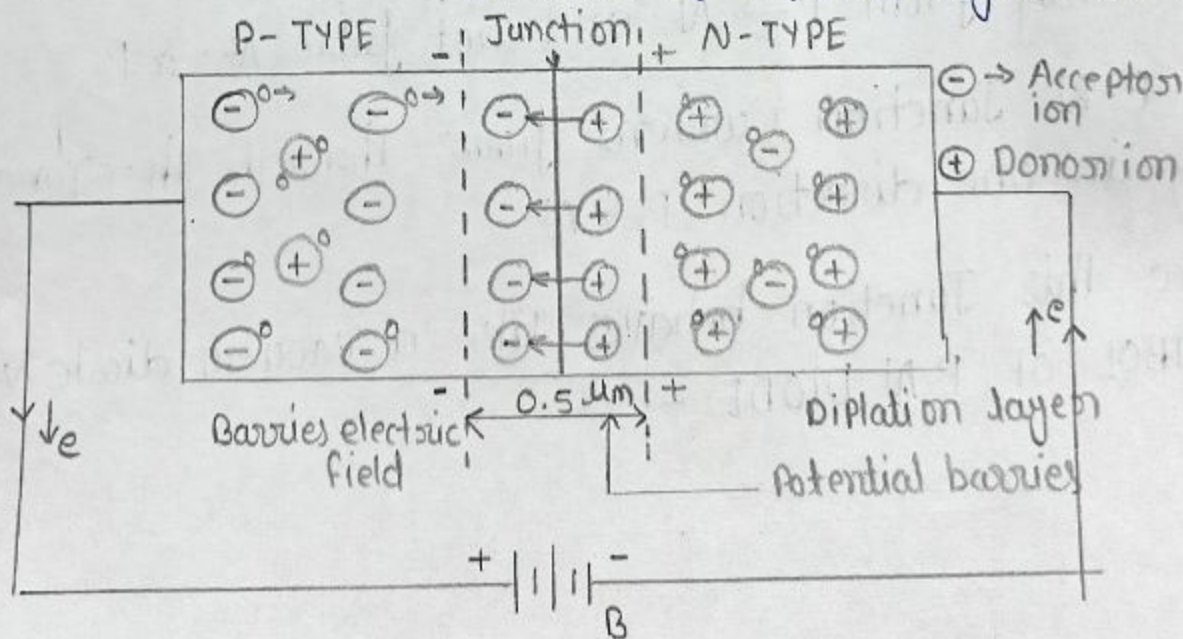
Characteristics of a P-N Junction

- i) Forward biased Condition
- ii) Reverse biased condition

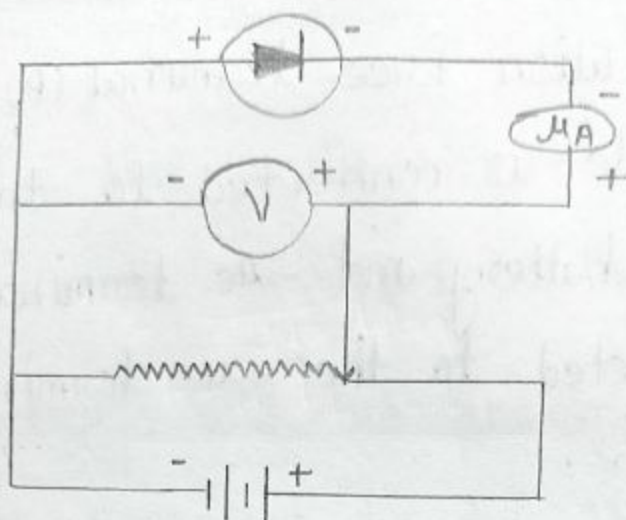


Forward bias condition:-

- ① Current I - high
- ② Resistance R - Low
- ③ Width of depletion layer $\downarrow$ (decreases)

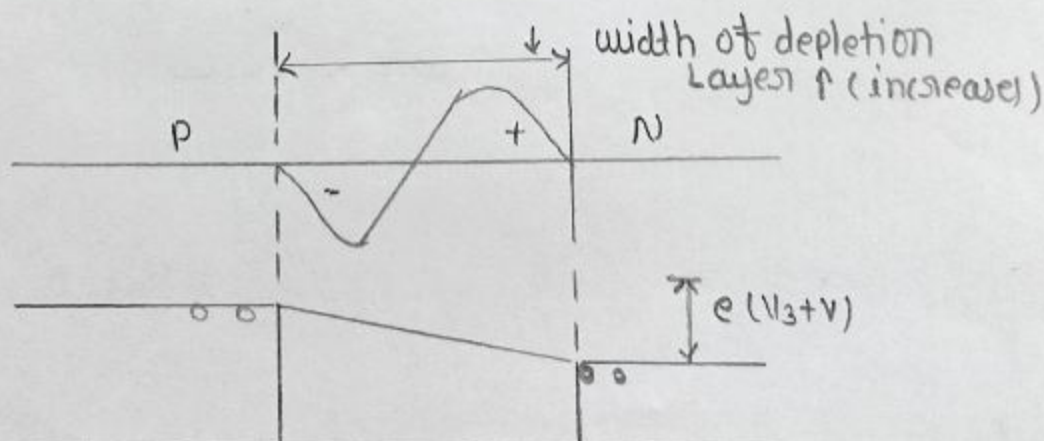
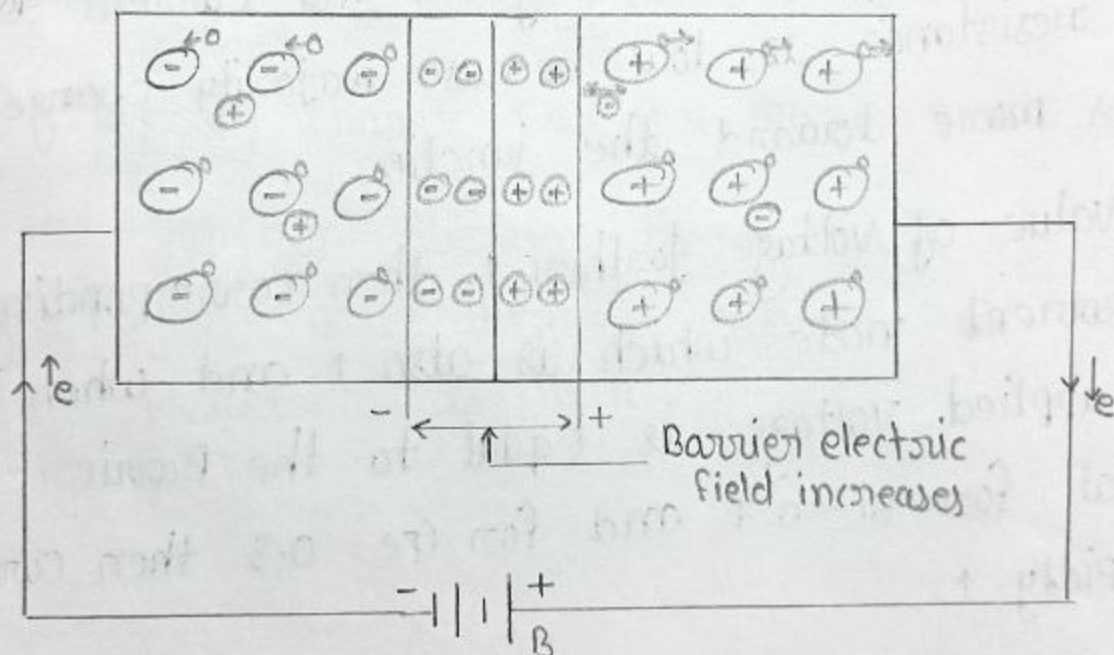


Reverses Biased Condition



In Reverse

1. Current I - Low
2. Resistance R = high
3. Width of depletion Layer $\uparrow$



Forward Biased Condition :-

In this condition free terminal (P) of the P-N Junction diode is connected to the free terminal of the battery and -ve terminal of the diode N is connected to the -ve terminal of the battery.

Then width of depletion Layer $\downarrow$ and current is high resistance is low because majority charge carries move toward the junction

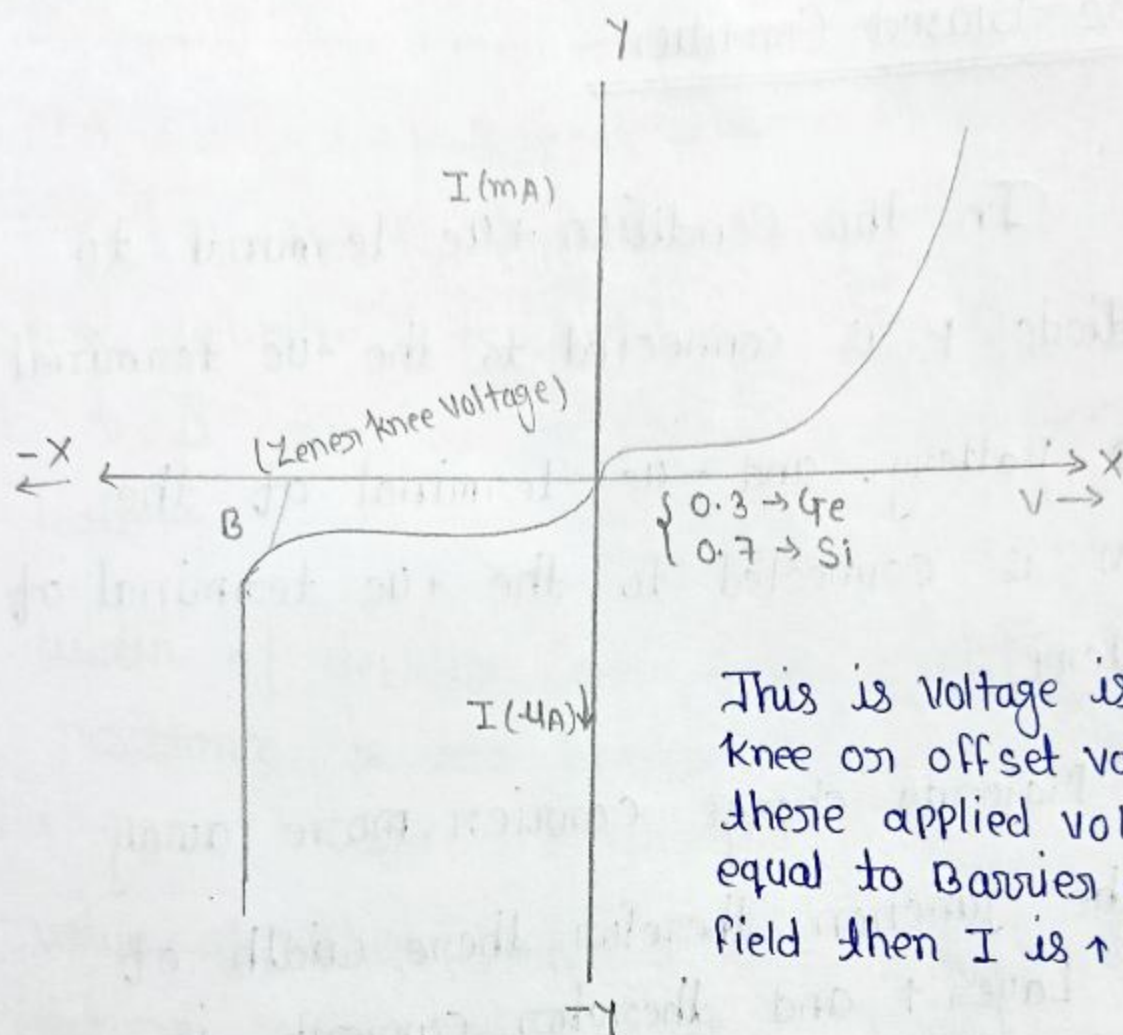
The value of voltage further $\uparrow$ then corresponding to current note which is also $\uparrow$ and when the applied voltage is equal to the Barrier Potential for Si - 0.7 and for Ge = 0.3 then current is rapidly $\uparrow$

Reverse Biased Condition:-

In this Condition +ve terminal to the diode p is connected to the -ve terminal of the Battery. and -ve terminal of the diode n is connected to the +ve terminal of the Battery.

Then Majority charge carrier move away from the junction therefore the width of depletion Layer $\uparrow$ and therefore current is low and Resistance is high.

Characteristic Curve :-



This is voltage is called knee on offset voltage. When applied voltage is equal to barrier electric field then I is $\uparrow$ rapidly.

REFERENCES:-

SUMMARY:-

TOPIC :- Zener Diode, Solar Cell.

OBJECTIVES:-

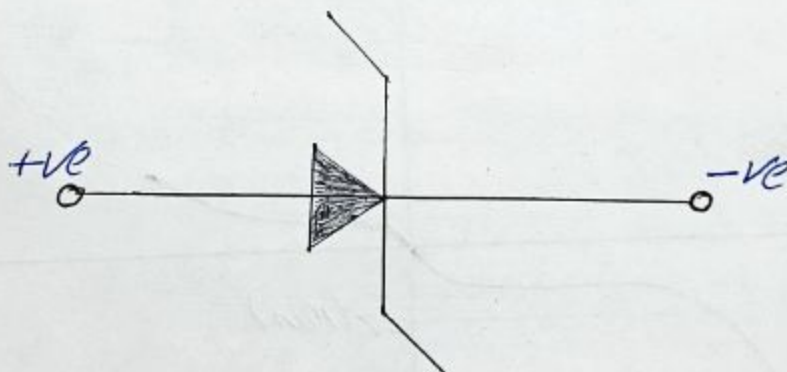
OUTCOME:-

Zener diode:-

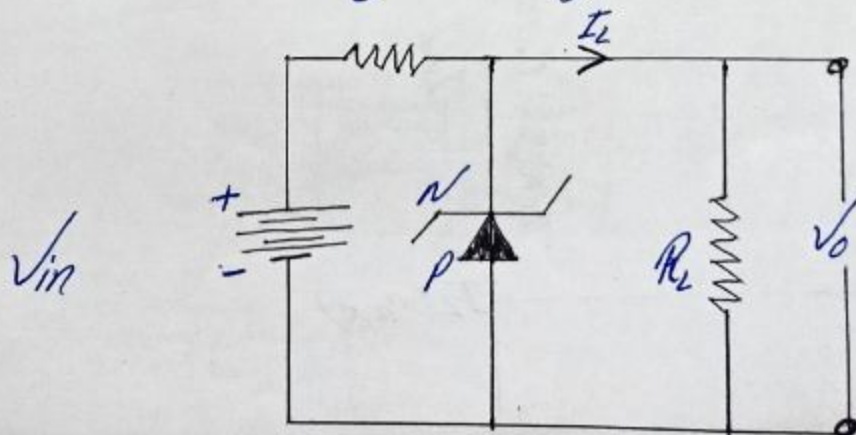
→ A heavily doped P-N junction diode is called zener diode.

It is doped operated at reverse bias condition on mode.

The Symbol of zener diode is.



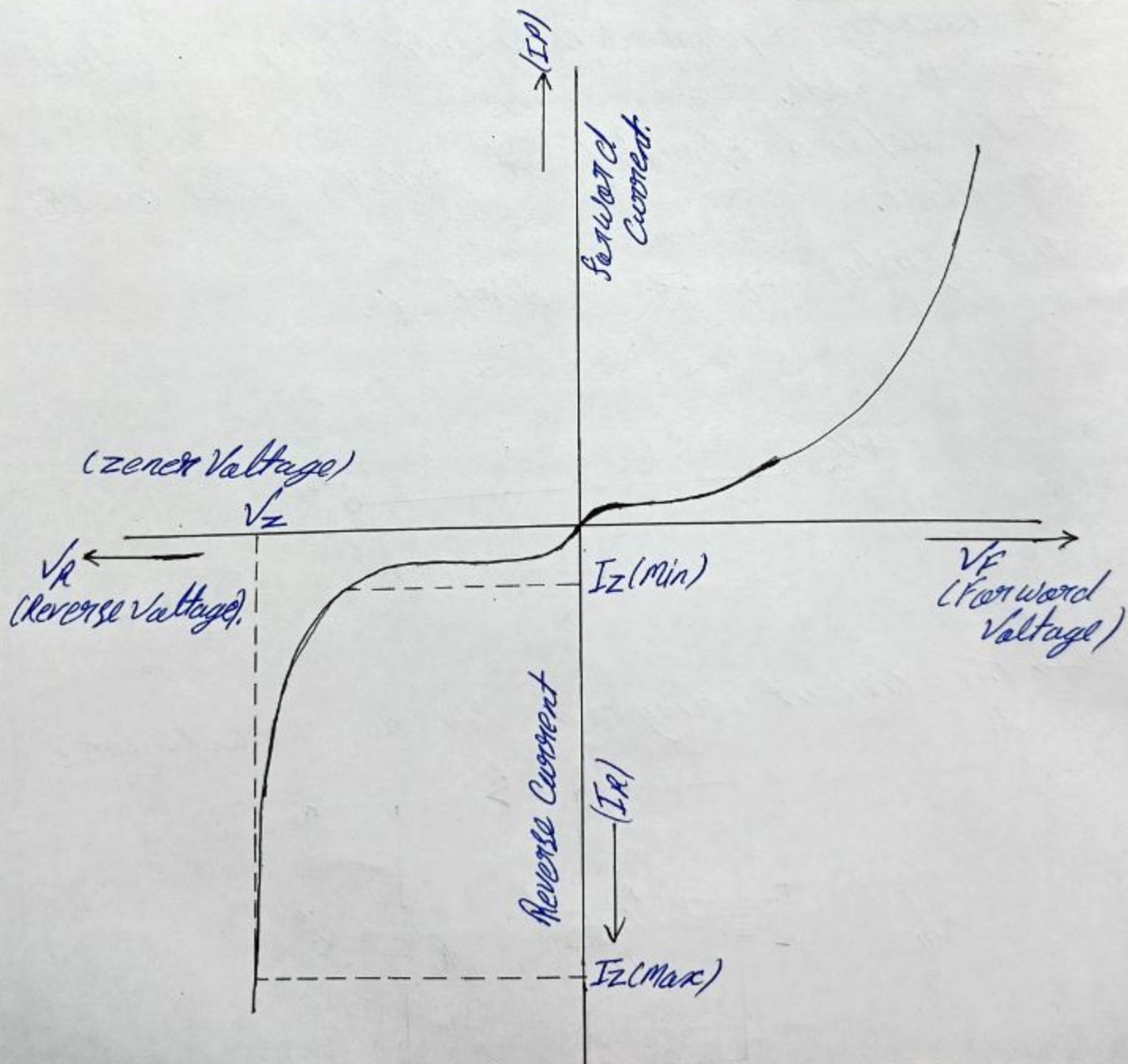
→ The circuit diagram for the characteristic is



→ In forward characteristic's of zener diode is similar to P-N Junction diode.

When zener diode is operated at reverse bias mode, then after break down voltage, voltage across zener remains constant.

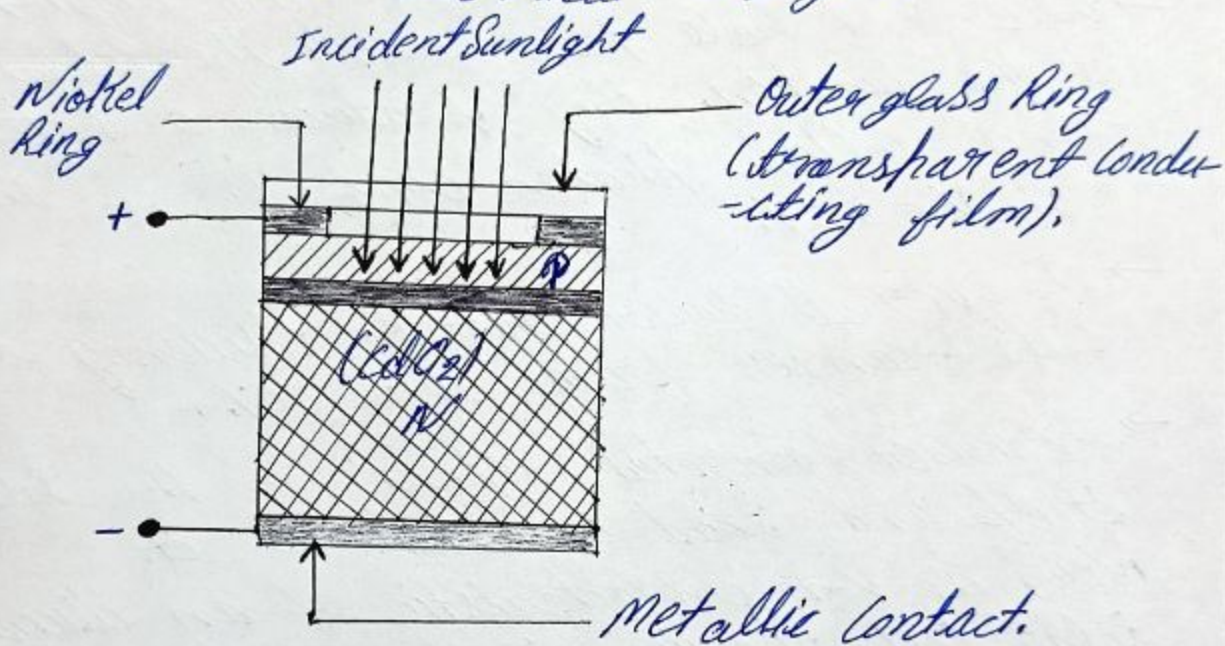
Hence, it act as a voltage regulator in Reverse bias mode.



Solar Cell :-

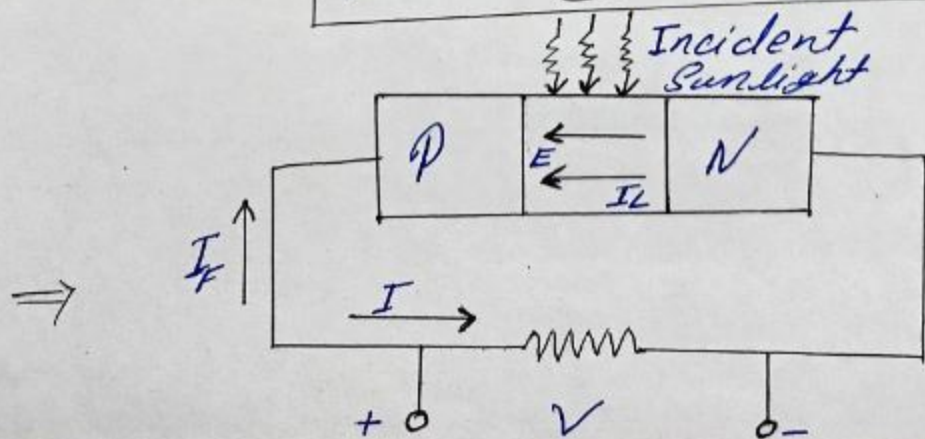
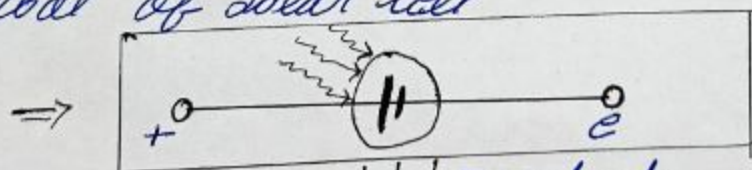
→ Solar Cell is basically a P-N junction diode that can generate electrical power when illuminated.

→ Solar Cell is a large area device illuminated with sunlight and it is convert the solar energy into the electrical energy's.



Solar Cell

Symbol of Solar cell



"Fig i-"

→ Shows a P-N junction solar cell with a resistive load R . When no bias is applied to junction an electric field E exists in the space charge. Let us consider that radiation are allowed to incident on the cell.

* Construction of solar cell:-
→ It consists of P-type diff on which a thin layer of n-type material is grown when solar light is incident on cell ends pairs are generated in N and P region.

→ The e^- and hole pairs reach the depletion region at the junction. they produces a potential difference called photon em.f is of the order of $V = 0.5V$.

→ The over all power conversion efficiency of solar cell's range's from 10 to 30%.

→ The Power generated depends on the solar cell itself and load connected to it.

Now, the V-I characteristic of an illuminated solar cell showing in fig. the point of Max^m power.

REFERENCES:-

SUMMARY:-

TOPIC :- Hall Effect :-

OBJECTIVES:-

OUTCOME:-

Hall Effect →

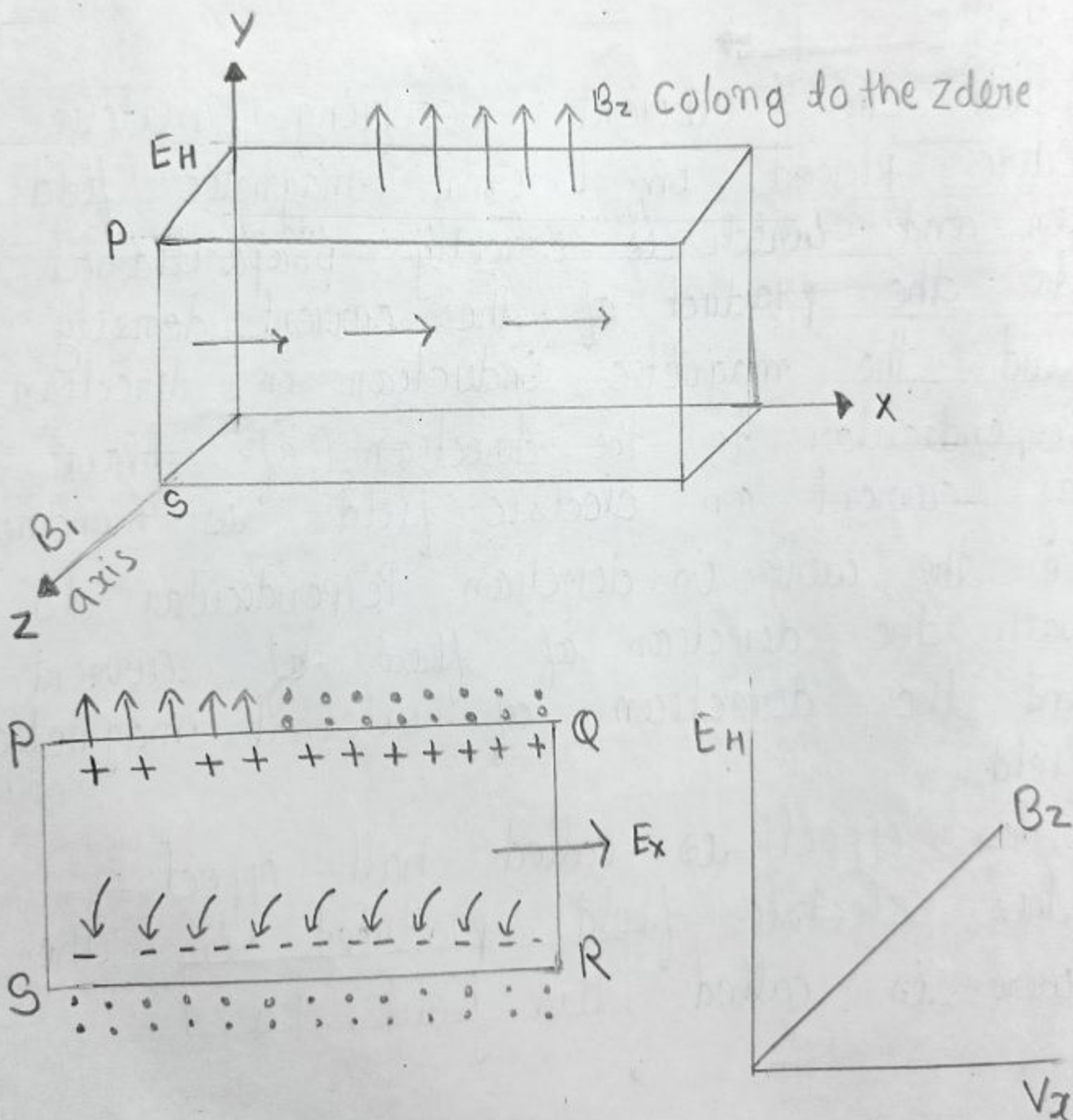
When the current carrying metallic wire is placed in external magnetic field and which is directly proportional to the product of the current density and the magnetic induction in direction perpendicular to the direction of flow of current an electric field is induced in the wire in direction perpendicular to both the direction of flow of current and the direction of external magnetic field.

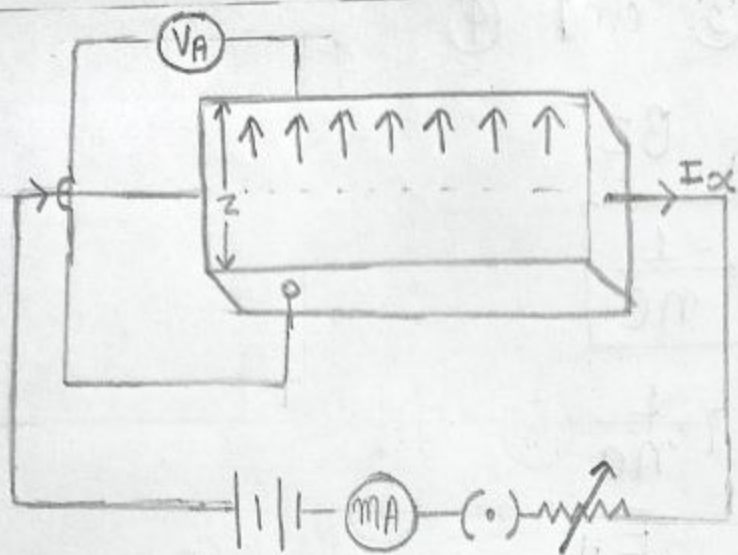
This effect is called hall effect.

This electric field produced in the wire is called Hall field (E_H).

and the potential difference developed at the end of the wire is called

Hall Voltage (V_H)





the Hall Voltage and Hall Coefficient
force on charge particle due to magnetic field

$$F = f_1 = -eE_H \quad \text{--- (1)}$$

force on charge particle due to magnetic field

$$f = f_2 = -e(V_x \times B_z) \quad \text{--- (2)}$$

from eqⁿ (1) and (2)

$$-eE_H = e(V_x \times B_z)$$

$$\boxed{E_H = V_x B_z} \quad \text{--- (3)}$$

current density along to x direction

$$I_x = -enV_x$$

where n is total charge density in the conductor per unit area

$$\text{then } V_x = \frac{-I_x}{ne} \quad \text{--- (4)}$$

from eqⁿ ③ and ④

$$E_H = -\frac{J_x}{ne} B_z$$

$$\boxed{\frac{E_H}{J_x B_z} = -\frac{1}{ne}}$$

$$\rightarrow R_H = -\frac{1}{ne}$$

$$\text{where } R_H = \frac{E_H}{J_x B_z} \quad \text{--- (5)}$$

let * the term (E_H) is Hall potential difference (V_H)

But $E_H = \frac{V_H}{y}$ [where y is the thickness of the bar]

and current density

REFERENCES:-

SUMMARY:-

$$J_x = \frac{I_x}{\text{area}} = \frac{I_x}{yz} \left[J_x = \frac{\text{current}}{\text{area}} \right]$$

(7)

z is width of the bar

from eqn (5) (6) and (7)

$$R_H = \frac{V_H}{y} \cdot \frac{1}{\frac{I_x}{yz} \cdot B_z}$$

$$\Rightarrow \frac{V_H y \cdot z}{I_x y B_z}$$

$$R_H \Rightarrow \frac{V_H z}{I_x B_z}$$

This is the Hall coefficient
and Hall voltage

$$V_H = \frac{R_H I_x B_z}{z}$$

Applications: of Hall Voltage

1. Hall effect can be used for the measurement of the strength of magnetic field
2. it is used for the determination of carrier concentration of semiconductors
3. the sign of Hall coefficient tells about the Predominant charge carriers in a semiconductor
4. Hall effect can be used for the amplification of weak signals

TOPIC :- Energy Band in Solid or Band theory

OBJECTIVES:-

OUTCOME:-

ENERGY BAND IN SOLID ORBAND THEORY

In Case of a single isolated atom there are single energy levels for example in case of hydrogen atom or in case of a gas where the atoms are sufficiently far apart not to exert any influence on one another.

But in case of solids, the atoms are arranged in a systematic space lattice and hence the atom is greatly influenced by neighbouring atoms. The closeness of atoms results in the intermixing of electrons of neighbouring atoms. As a result the valence electrons in the outermost shells which are not strongly divided by nuclei.

Due to intermixing the number of permissible energy levels increases or there are significant changes in

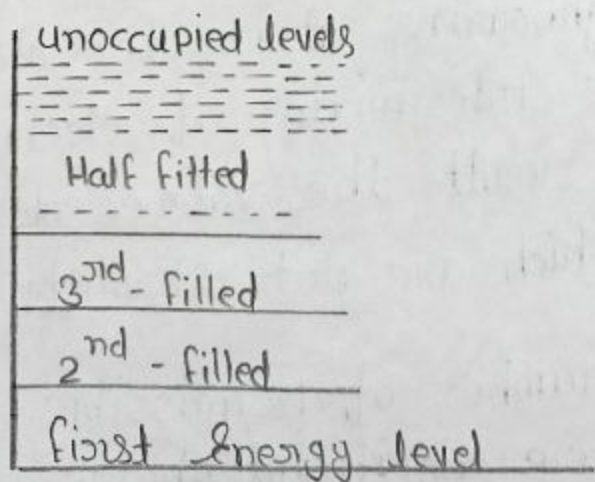
The Energy levels.

"In case of a solids instead of single energy levels associated with the single atom, there will be bands of energy levels. A set of such closely packed energy level is called an energy band.

"In case of a solid Hence the bands of energy levels are referred to the entire solid and not to the single individual atom, the energy bands may be continuous.

The concept of energy bands given by the energy level diagram.

- 1) The energy level of a single isolated atom of Silicon, Here an electron can have only single energy corresponding to the orbit in which it exists.



a. Energy level of a single isolated atom Now in each Silicon atom has $14 e^-$ two of which occupy the M shell. and K shell occupy L shell.

We know that two electron of M shell are in sub shell 3s and two are in sub shell 3p i.e. the 3p sub shell is half-filled because it can accommodate a total of 6 electrons Hence 3d sub-shell is unoccupied

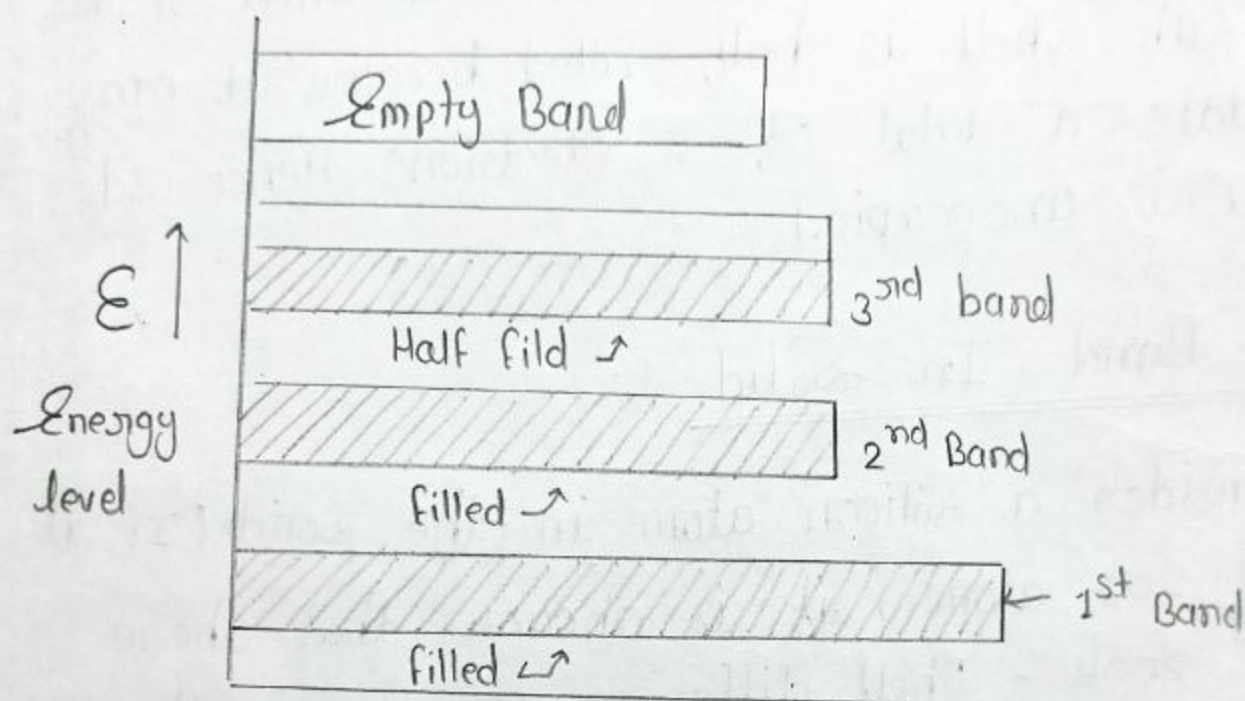
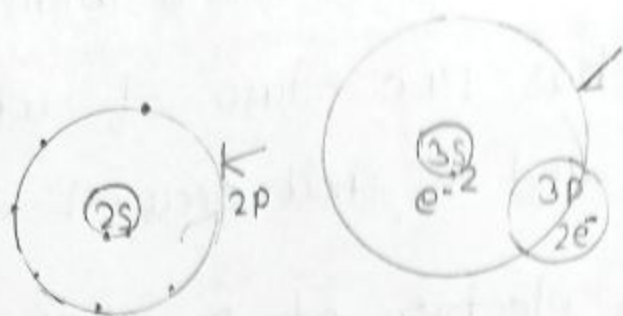
Energy Band In Solid

Now consider a Silicon atom in the solid (If it containing 200 atoms) it is obvious that there would be 200k-shell differing in energy by extremely small amounts the reason being that no. to electrons in this shell. See exactly the same charge environment.

The 200 k-shells differing in extremely small amounts of energy are so close as to merge into one energy band. This is called first band.

Shown in figure —

Similarly, second orbit electrons form second energy band and so on —



TOPIC :- Kronig Penny Model

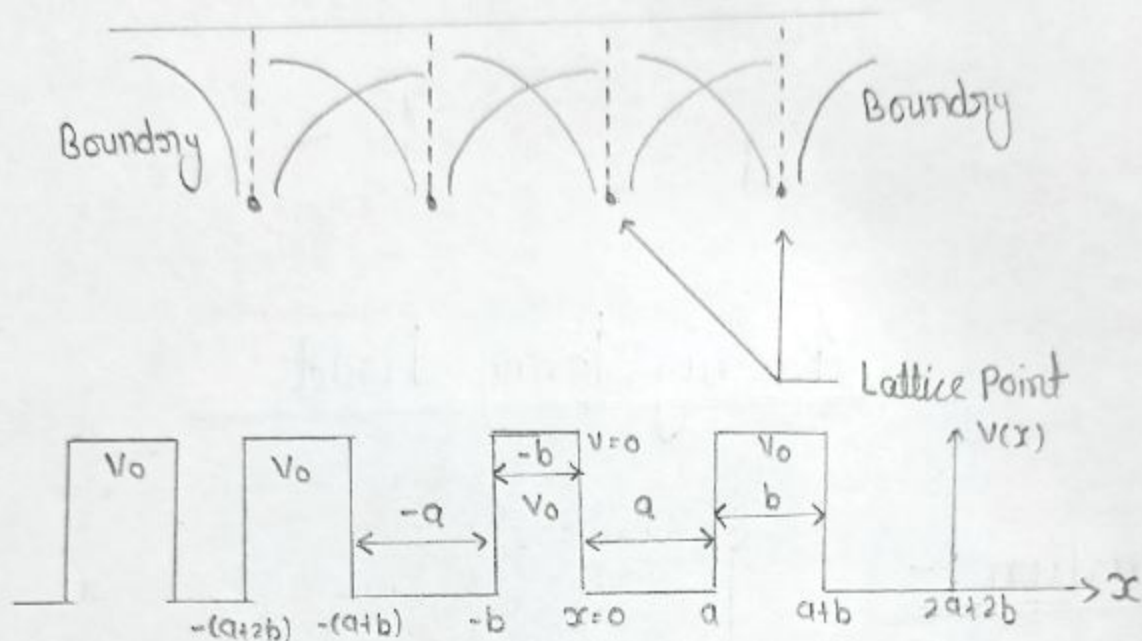
OBJECTIVES:-

OUTCOME:-

Kronig Penny ModelExplanation :-

Kronig and Penny solved the Schrodinger Equation for a particle in a periodic potential in a crystal lattice.

They solve the potential problem i.e. potential energy of electron has the form of rectangular barrier. which is shown in figure.



From figure Region $0 < x < a$, $(a+b) < x < (2a+b)$,

$V(x) = 0$ because particle inside

and Region $-b < x < 0$, $-(a+2b) < x < -(a+b)$,

Potential Energy is constant which is $V(x) = V_0$

we know that the Schrodinger Equation

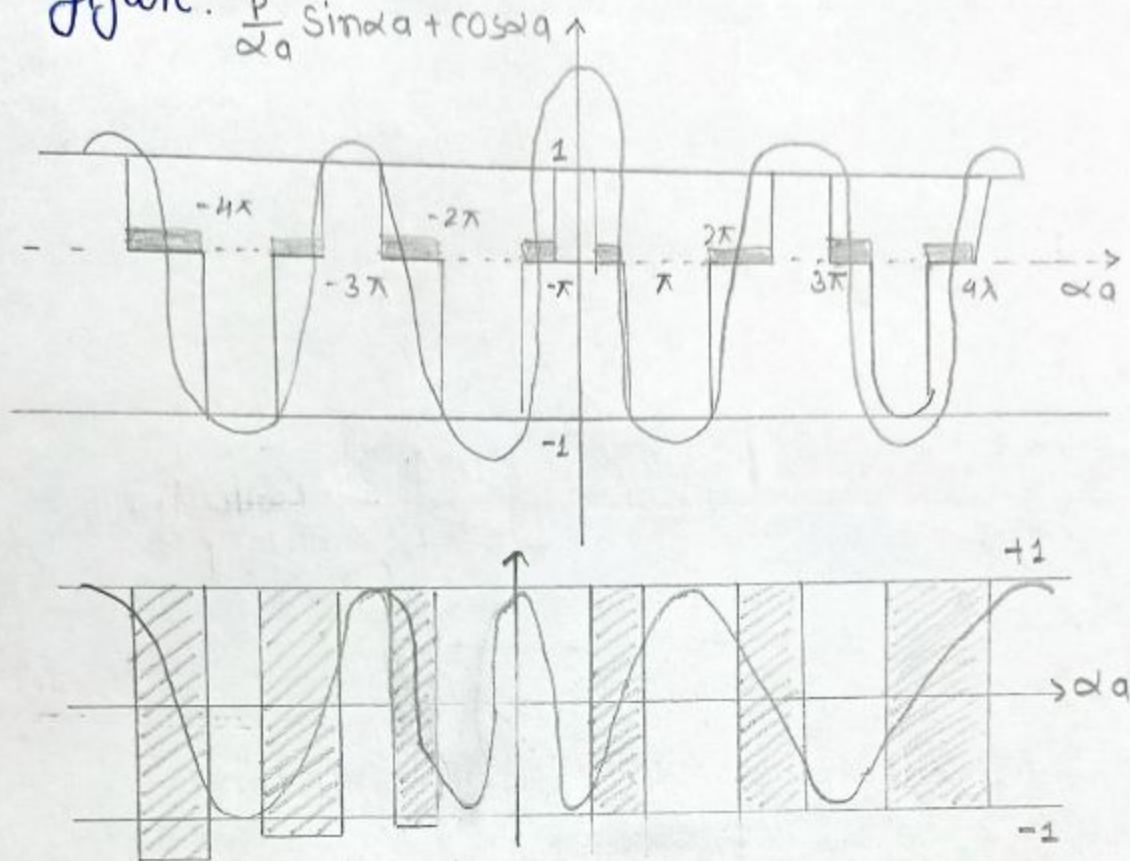
$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (\epsilon - V) \psi = 0$$

The Solution of this Equation for Each interval

is $\frac{P}{\alpha a} \sin \alpha a + \cos \alpha a = \cos k a$

where $P = 3\pi/2$ and $\alpha^2 = \frac{2m\epsilon}{\hbar^2}$ or $P = m v_0 a b / \hbar^2$

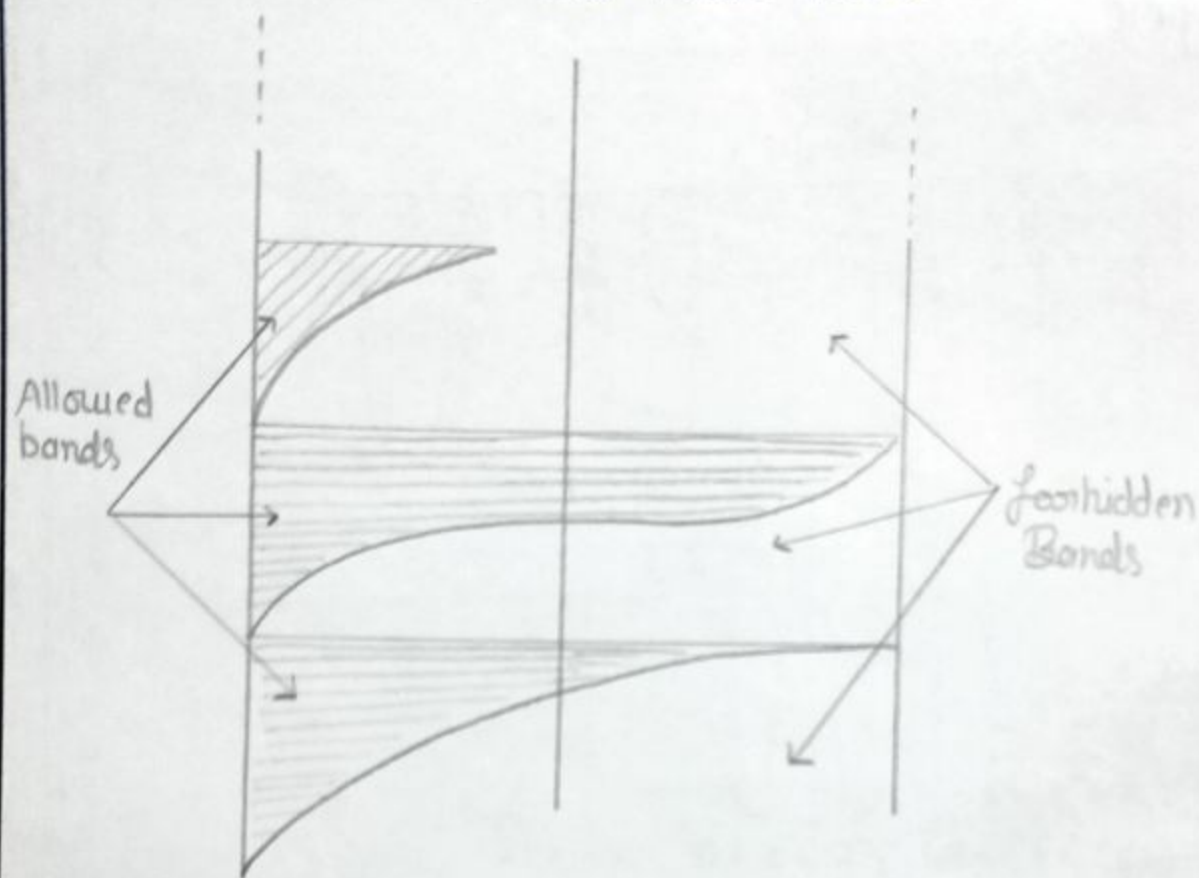
Then graph between $P/\alpha a \sin \alpha a + \cos \alpha a$, αa shown in figure.



following conclusion on the basis of the theory

- The allowed energy bands cover these regions of curve which is b/w value +1 and -1. There are allowed bands separated by forbidden bands the allowed energy bands are represented in the fig. by dark thick lines and forbidden bands by them dashed lines.
- The boundaries of the allowed energy bands correspond to the values of $\cos ka = \pm 1$ when $ka = n\pi$, $k = \frac{n\pi}{a}$
 a is crystal lattice width
- width of the allowed energy bands $\uparrow$ from the origin while width of forbidden energy bands $\downarrow$
- When $P = \frac{m v_0 a b}{\hbar^2}$, the value of $P \uparrow$ then e^- is bound more strongly to the potential well, and value of $P \downarrow$ then the

e^- becomes more and more free.



REFERENCES:-

SUMMARY:-

TOPIC :- Laser and its properties :-

OBJECTIVES:-

OUTCOME:-

LASER

LASER :- Light Amplification by Stimulated emission of Radiation. It is developed in 1950

Definition of Laser:-

Laser as a beam of light monochromatic coherent, unidirectional and highly intensified.

It is a device to produce a strong monochromatic, coherent, unidirectional and highly intensified beam of light.

Principle of Laser:- It is based on the principle of stimulated emission.

PROPERTIES OF LASER

1. Monochromatic
2. Coherent
3. Directionability
4. Intensity
5. Divergence

1. Monochromatic :- Extraordinary monochromatic light from a laser beam. While light from an ordinary source is never monochromatic. The light from normal monochromates source spreads over a wavelength range of the order of $100 \text{ \AA}$ to $1000 \text{ \AA}$ on the other hand, in case of laser the spread is of the order of a few angstroms only have single colour. $\checkmark$, λ

2. Coherent :- The laser beam is completely high coherent it is possible to observe interference effect from two independent laser beams.
i.e. coherent, two source of light said to be coherent will they emit continuously beam of light have same $\checkmark$, λ and

phase difference constant with the time.

3. High directionability :- An ordinary source of light radiates light in all directions. On the other hand a laser source emits radiation only in one direction. i.e. a laser beam is highly directional. High directionability of laser radiation is of special significance in advanced researches.

4. High Intensity :- The intensity of light is defined as the energy passing normally per unit area per second through a point normal to the direction of flow. For an ordinary spherical source at a distance r , the intensity I is given by -

$$I = \frac{P}{4\pi r^2}$$

where, P is power of the source. In case of laser beam, the energy is concentrated in a very small region.

For example,

1 W laser source will appear many

times more intense than an ordinary 100 W source.

5° Divergence :- The light from conventional source spreads out in the form of spherical wavefront. Hence they are highly divergent, The divergence or angular spherical of the laser beam is extremely small.

REFERENCES:-

SUMMARY:-

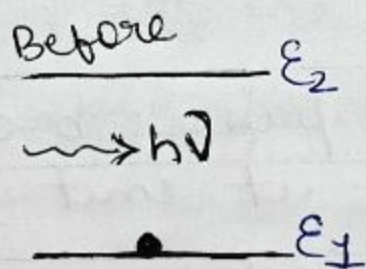
TOPIC :- Absorption, and Emission

OBJECTIVES:-

OUTCOME:-

ABSORPTION EMISSION

If an atom is initially in a lower energy level (i.e. in a ground energy state) if the photon of energy $h\nu$ ($E_2 - E_1$) is incident on the atom it absorb a Quanta of radiation then the atom get excited energy state. This condition is known as Absorption emission.



The absorption transition T_{12} ($1 \rightarrow 2$) depends on the properties of the energy state one & two is proportional

to the energy density $u(\nu)$ of the radiation at frequency ν . Incident the atom the rate of absorption is

$$P_{12} \propto u(\nu)$$

$$P_{12} = B_{12} u(\nu)$$

where Einstein B coefficient (B_{12})

where B_{12} is proportionality constant & known as Einstein B coefficient

$u(\nu)$ is the photon density of the incident beam

SELF EMISSION

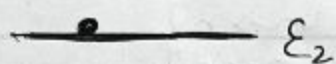
If an atom is initially in a ground energy state an external energy is imparted to the substance its atom absorb the energy & rises to high energy state since atom can't stay for a longer time in a time of about 10^{-10} sec.

The atom passes back to its ground state it emit photon having energy $h\nu = (E_2 - E_1)$

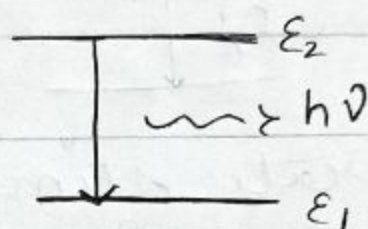
Emission of photon without any external energy is called the self emission. It depends upon

only properties of E_2 & E_1 and Independent on the photon density. Thus transition one is equal to the Einstein Coefficient of

$$P_{21} = A_{21} \quad \text{spontaneous radiation}$$



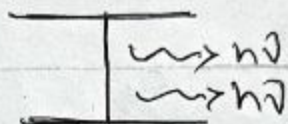
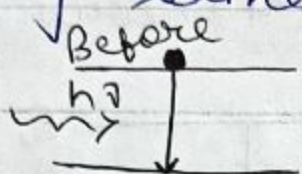
$$P_{21} = A_{21}$$



STIMULATED EMISSION

If the excited state of the atom, a photon of frequency same as the frequency of photon which the atom is passing from the excited state to the ground state will emits strikes the atoms then that atom comes to the ground state very smooth and stimulate the light incident on it, by emitting the photon of exactly the same frequency that is called stimulated emission.

It stimulated emis. The light emitted from each atom is coherent, unidirectional & monochromatic and phase is exactly same as that of incident photon.



The rate stimulated emission

$$P_{21} \propto \nu(\nu)$$

$$P_{21} = B_{21} \nu(\nu)$$

stimulated emission is essential for laser action because in this photon an intense light beam is obtained by amplifying the light by stimulated emission of radiation.

REFERENCES:-

SUMMARY:-

TOPIC:- Relationship b/w Einstein "A" and "B"

OBJECTIVES:- Coefficient

OUTCOME:-

RELATIONSHIP b/w EINSTEIN A & B COEFFICIENT

Let two energy state E_1 and E_2 and N_1 and N_2 no. of atom in ground and excited energy state.

Atom in thermal equilibrium at temperature T & radiation of frequency ν . Since the probable rate of absorption of radiation $u(\nu)$ the no. of transition per unit time and per unit volume from E_1 to E_2

$$N_1 P_{12} = B_{12} N_1 u(\nu) \quad \dots \dots (1)$$

N_1 is the no. of atoms in energy state E_1 .

Similarly the no. of transition (self or spontaneous and stimulated emission) per unit time & per unit volume from $E_2 \rightarrow E_1$

$$N_2 P_{21} = N_2 A_{21} + N_2 B_{21} u(\nu) \quad \dots \dots (2)$$

In Equilibrium state.

i.e. the no. of atoms absorbing radiation per unit time is equal to the no. of atoms emitting radiation per unit time.

$$N_1 P_{12} = N_2 P_{21}$$

$$B_{12} N_1 U(\nu) = N_2 A_{21} + N_2 B_{21} U(\nu)$$

$$(B_{12} N_1 - N_2 B_{21}) U(\nu) = N_2 A_{21}$$

$$U(\nu) = \frac{N_2 A_{21}}{(B_{12} N_1 - N_2 B_{21})} = \frac{N_2 A_{21}}{N_2 B_{21} \left(\frac{B_{12} N_1}{N_2 B_{21}} - 1 \right)}$$

$$U(\nu) = \frac{A_{21}}{B_{21} \left(\frac{B_{12} N_1}{N_2 B_{21}} - 1 \right)} = \frac{A_{21}}{B_{21} \left(\frac{N_1}{N_2} \frac{B_{12}}{B_{21}} - 1 \right)}$$

Einstein Proved Thermodynamically

$$B_{12} = B_{21}$$

By putting this in above we get

$$U(\nu) = \frac{A_{21}}{B_{21} \left(\frac{N_1}{N_2} - 1 \right)} \quad \dots \dots \dots (3)$$

According to Boltzmann distribution

$$N_1 = e^{-\epsilon_1/KT} \quad \text{and}$$

$$N_2 = e^{-\epsilon_2/KT}$$

$$\frac{N_1}{N_2} = \frac{e^{-\epsilon_1/KT}}{e^{-\epsilon_2/KT}} = e^{-\epsilon_1/KT} \cdot e^{+\epsilon_2/KT} = e^{(\epsilon_2 - \epsilon_1)/KT}$$

$$\frac{N_1}{N_2} = e^{h\nu/KT}$$

Put $\frac{N_1}{N_2}$ in eqⁿ (3)

$$u(\nu) = \frac{A_{21}}{B_{21}} (e^{h\nu/KT} - 1) \dots (4)$$

But According to Planck energy density at radiation of frequency ν & temperature T .

$$u(\nu) = \frac{8\pi \nu^3 h}{c^3} \frac{1}{(e^{h\nu/KT} - 1)} \dots (5)$$

on Equation (4) & (5) we get

$$\frac{A_{21}}{B_{21} (e^{h\nu/KT} - 1)} = \frac{8\pi \nu^3 h}{c^3} \cdot \frac{1}{(e^{h\nu/KT} - 1)}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi \nu^3 h}{c^3}$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}}$$

This is the Relationship between Einstein A & B Coefficient.

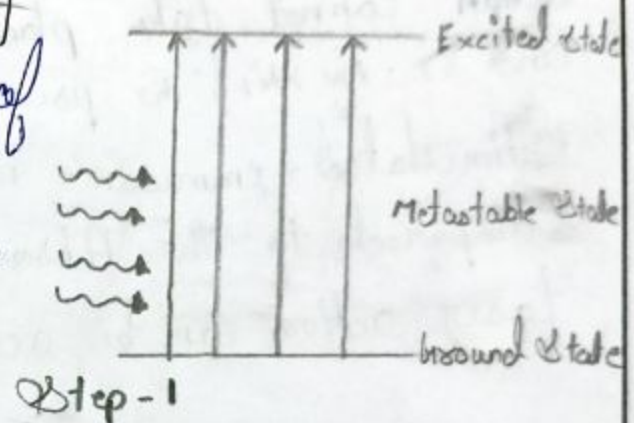
TOPIC :- Population Inversion and pumping with Laser Action

OBJECTIVES:-

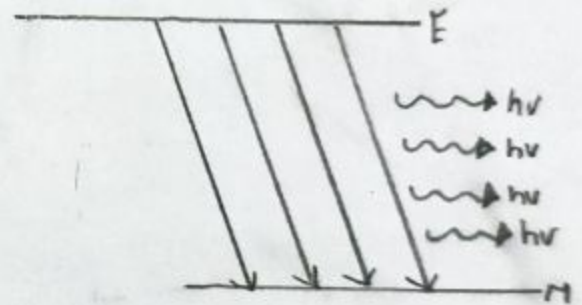
OUTCOME:-

* Population Inversion & pumping with Laser Action.

The State in which the population of atom in a higher State is larger than that in the ground State is known as population inversion
i.e. $N_2 \gg N_1$



Thus, when general condition of Population ($N_1 > N_2$) is inverted the population inversion ($N_2 - N_1$) is achieved.

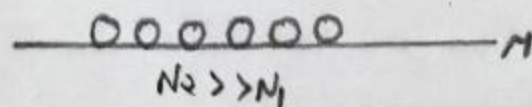


According to Boltzman eqn if N_1 and N_2 no. of atom in ground & excited energy State is (Steps).

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

$$\frac{N_2}{N_1} = e^{-h\nu/kT}$$

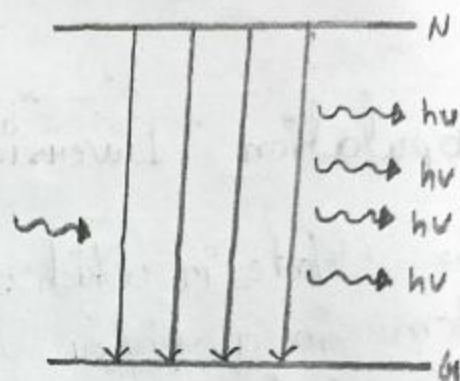
Step-2



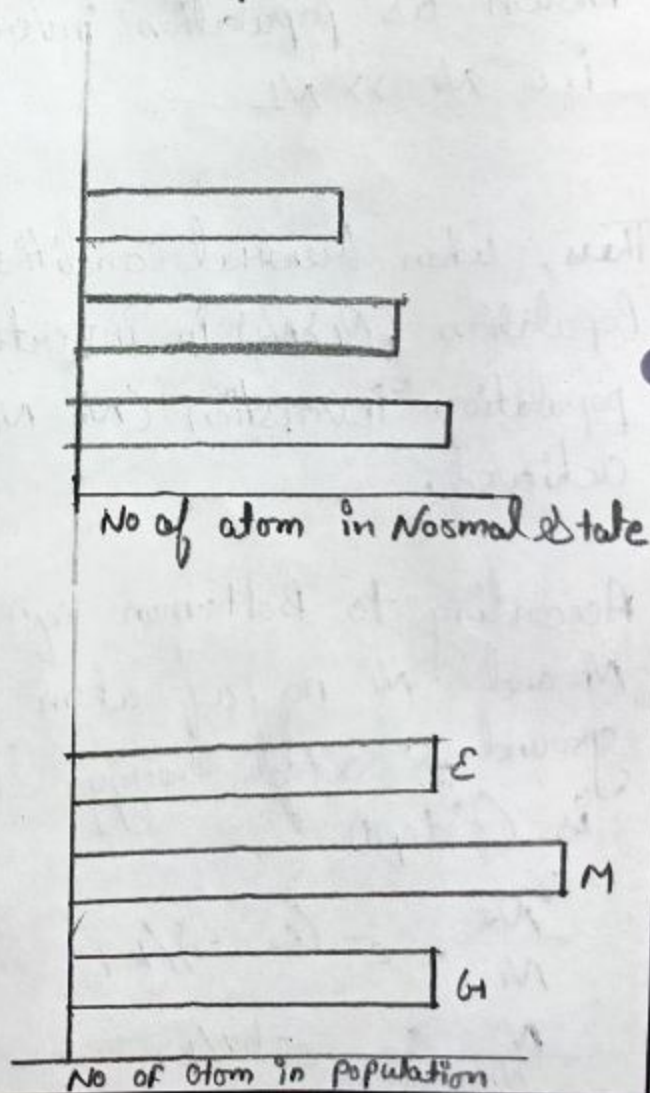
In equilibrium the population of N_2 State is very much smaller than the ground State. i.e. $N_1 > N_2$

Step-3

As a result the no of Stimulated emission are very little compared to the Absorption. Emission ~~action~~ cannot take place. and if $N_2 > N_1$, The process of Stimulated emission is more compared to the Absorption. Laser action can be achieved

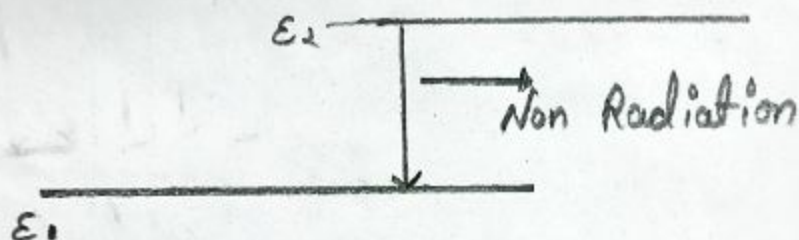
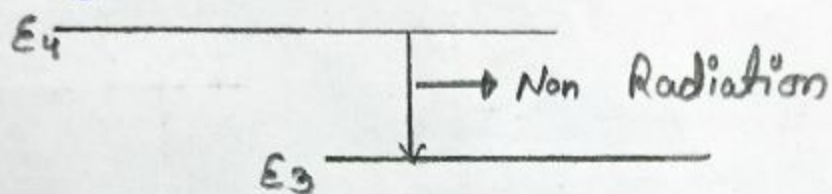


Step-4



pumping.

The pumping of achieving population inversion is known as pumping of atom.



TOPIC :- Ruby Laser

OBJECTIVES:-

OUTCOME:-

Ruby laser :- It is a pulse laser in which population inversion is done with the help of strong pumping pulse (by xenon flash tube). Its main part are.

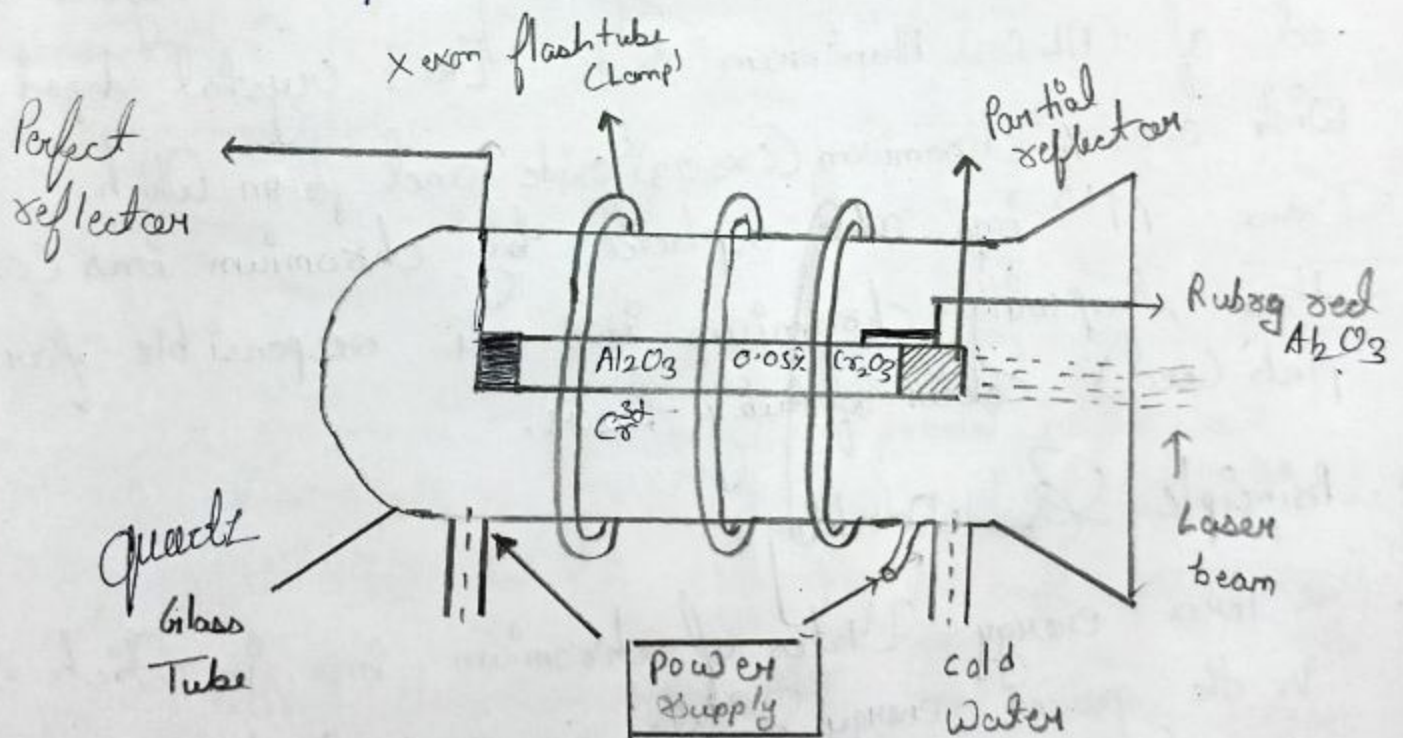


Fig (a)

→ Working :- It is a rod of Ruby crystal.

→ Resonance cavity :- The left end of the rod is perfect reflector & right end of the reflector is partial reflector both ends are perfectly plane and $\parallel$.

→ optical pumping :- The Ruby crystal rod is surrounded by a glass tube in which flowing cold water. The rod is kept cold.

Basically Ruby rod is a crystal doped of (Al_2O_3) Aluminium oxide [RC] crystal doped with 0.05% chromium (Cr_2O_3) oxide and form which.

Some Al^{+3} ions are replaced by chromium ions (Cr^{3+})

Thus, impurity chromium ions are responsible for pink (red) colour of Ruby laser.

* Principle & working.

- Three energy states of chromium ions in which E_0 is the ground energy state E_1 & E_2 is the excited energy state & M is Metastable states.

- An ion can remain in Metastable energy E_2 & E_1 is the excited energy state for period of about 10^{-3} sec. Where, as it can remain in the excited state for a period of about 10^{-8} sec.

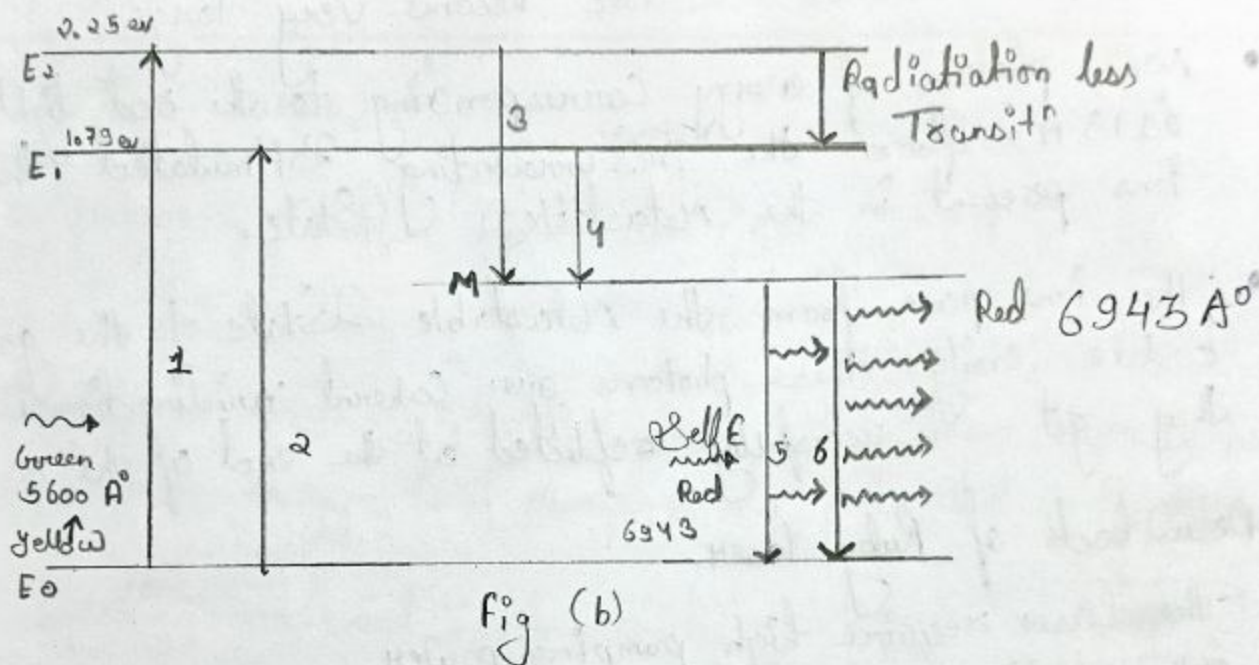


Fig (b)

- When the flash of light from the flash tube falls on the Ruby rod. The chromium ions present in the rod absorb green yellow light ($\lambda = 560 \text{ Å}$) & get excited to the energy states E_2 and E_1 transition 1 and 2 respectively.
- These excited chromium ion Cr^{3+} lose some energy in collision with the crystalline & come to the metastable state by the transition state 3 and 4.
- This process is repeated so that an intense unidirectional & coherent red light ($\lambda = 6943 \text{ Å}$) beam is

In pulse form have it can't be used for practically purpose

- Since the ions can remain in the metastable State M for a long period 10^{-3} Sec

∴ More & More ions comes to this State & the no. of ions in the metastable State become very large.

- Now, photons of energy corresponding to the red light $\lambda = 6943 \text{ \AA}$, from the surrounding stimulated the ions present in the metastable State.
- The ions pass from the metastable State to the ground State easily. These photons are coherent unidirectional and they get successfully reflected at the end of the rod.

Drawback of Ruby laser.

- I. The laser require high pumping power
- II. The efficiency of ruby laser is very small.
- III. The laser o/p is not continuous. The o/p occur in the form of pulses.

REFERENCES:-

SUMMARY:-

TOPIC :- Nd-YAG - Laser

OBJECTIVES:-

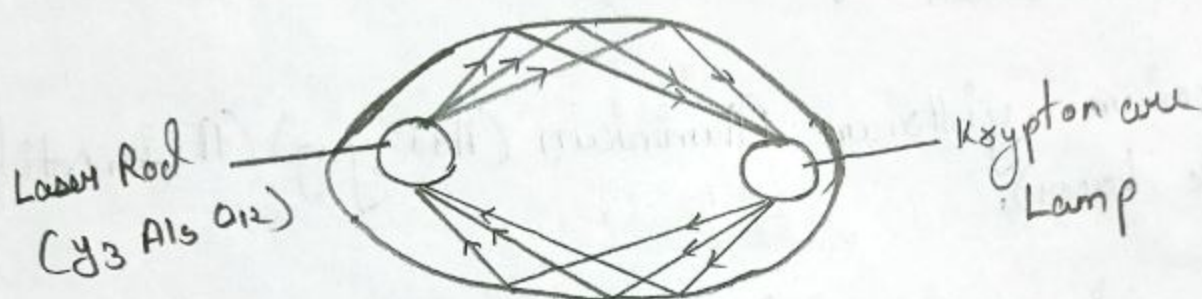
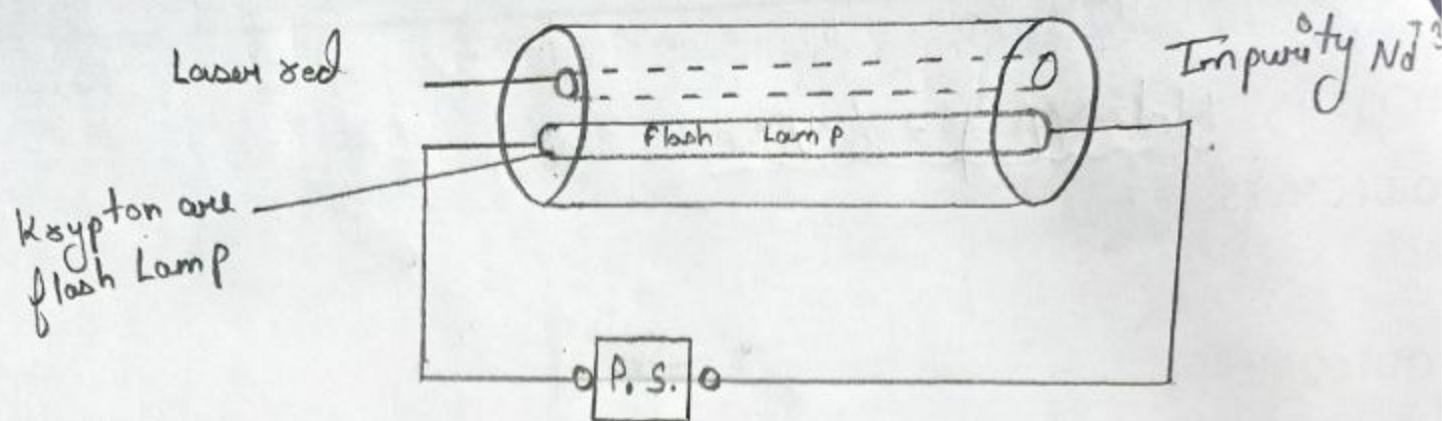
OUTCOME:-

(Nd-YAG - Laser) :-

Neodymium - yttrium Aluminium (Nd-YAG) (It is solid State Laser).

Nd-YAG is another Solid Laser in which Laser beam active by four energy level pumping its puts and continuous.

- I. Working material :- in which working substance is $\frac{1}{3}$ Al_2O_3 and its preparation by doping yttrium aluminium garnet ($\frac{1}{3}$ Al_2O_3) with neodymium ions (Nd^{+3}) (1%) Nd^{+3} ions are active for laser actions.
- II. optical pumping System :- or pumping source it is by krypton arc flash lamp also use
- III. Resonance cavity :- or optical resonator elliptical cylinder consist two parallel elliptical reflector one end is partially silvered and other end is perfectly silvered.

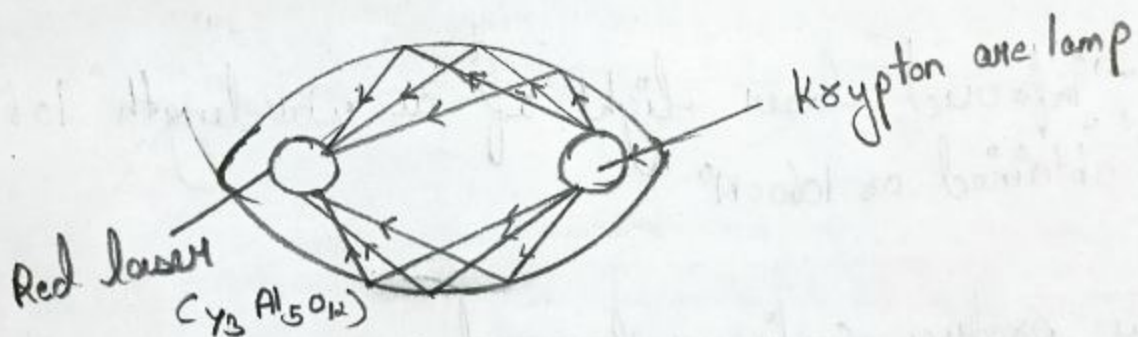
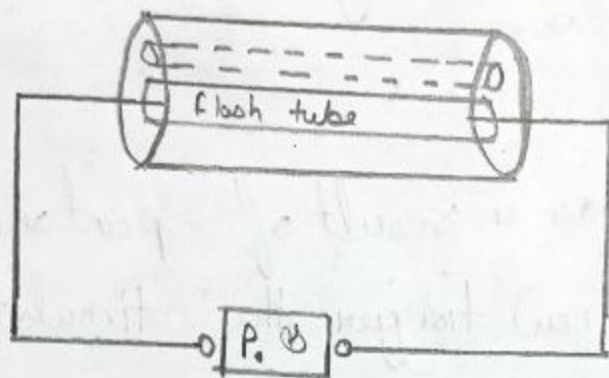


construction :- It consists elliptically cylinder has two parallel ellipsoidal reflector and focus lines. Along one focus line the laser the rod [$\text{Nd}^{3+} \text{YAG}$ rod] is fitted along another focus line.

principle :- Light emitted by flash lamp at one focus, after reflections the Silver Surface passes through the second focus.

i.e. whole of the flash lamp radiation get focussed on the $\text{Nd}^{3+} \text{YAG}$ rod,

Working :- Nd^{3+} ions are responsible for lasing action and this laser is four energy level.



- When light flash is incident on the laser seed Nd^{3+} ions are excited there light of wavelength range 5000 to 8000 $\text{\AA}$ absorbed by the Nd^{3+} are excited to higher energy states (band).
- The Nd^{3+} ions in the excited energy band rapidly decay to the level E_3 because there are about 10^{-8} sec.

- E_3 is metastable state therefore a population inversion state is reached in a very short time b/w states E_3 and E_2 (10^{-3}) Sec.
- Any photon emitted as a result of spontaneous emission from E_3 to E_2 can now trigger the stimulated process by $E_3 \rightarrow E_2$ transition.
- The infrared laser light of a wavelength $10640 \text{ \AA}$ is obtained or $10600 \text{ \AA}$.
- They produce continuous laser at room temp.

REFERENCES:-

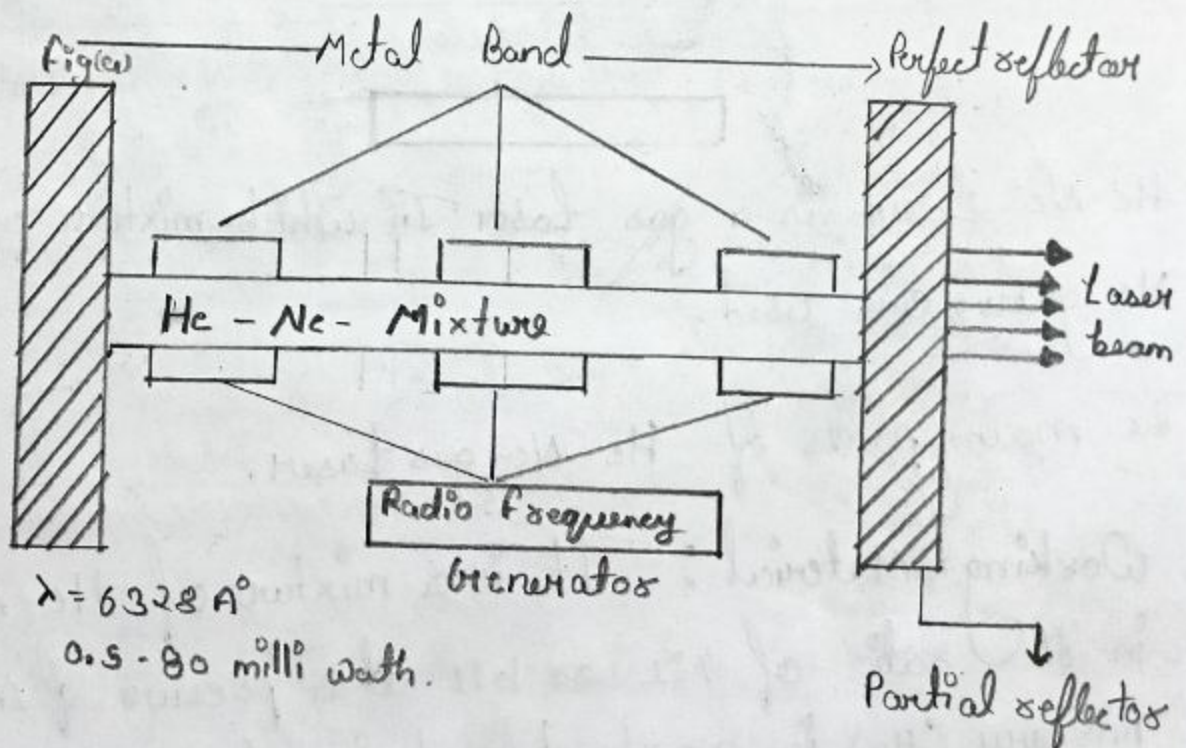
SUMMARY:-

TOPIC :- He-Ne-Laser

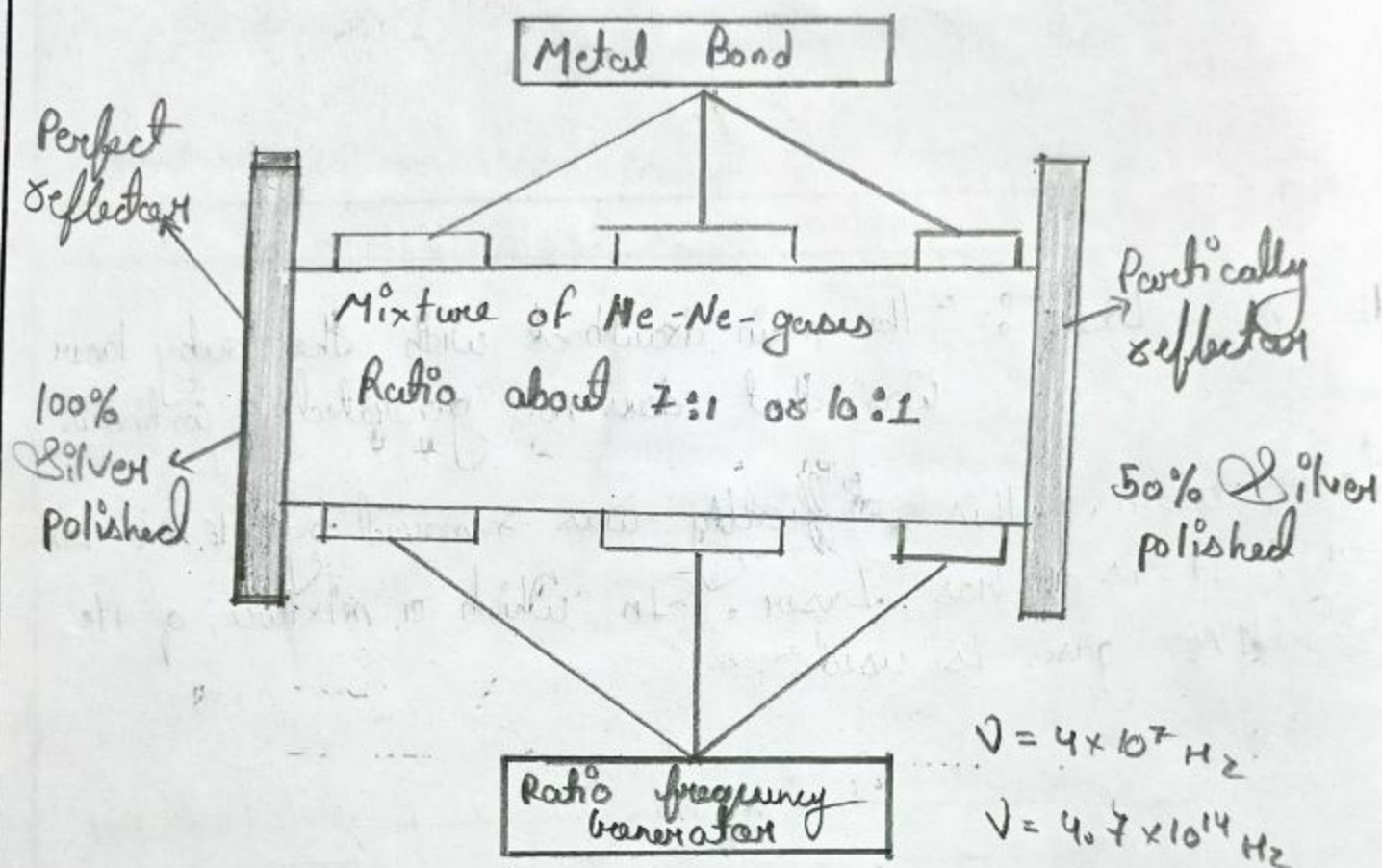
OBJECTIVES:-

OUTCOME:-

HE - NE - Laser :- The main drawback with the ruby laser was that does not generated a continuous laser beam. This difficulty was removed by HE-NE-laser. It is a Gas Laser. In which a mixture of He & Ne gases is used.



- Explain working of He-Ne laser
- Construction diagram of He-Ne laser



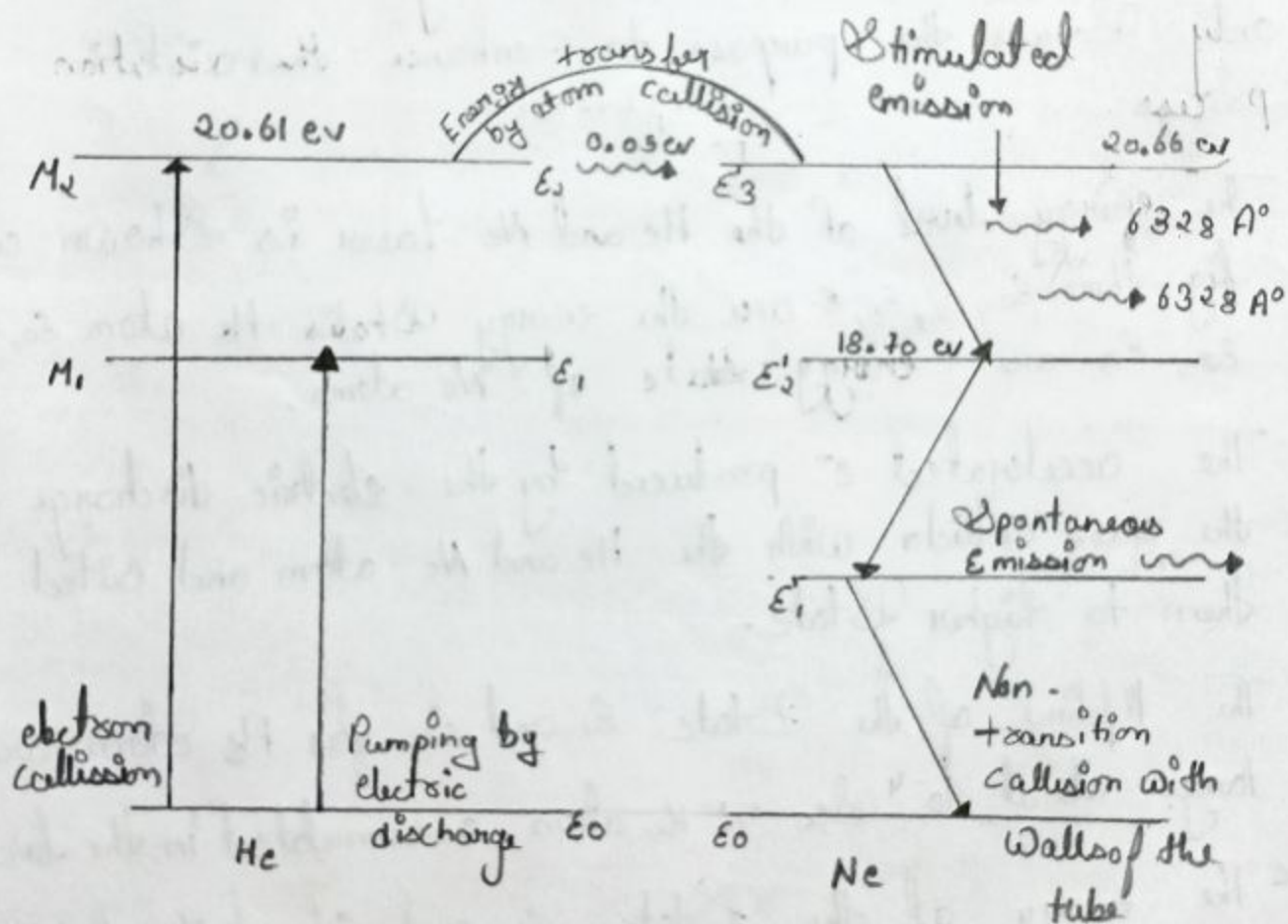
He-Ne Laser is a gas laser in which mixture of He and Ne gases are used.

The main parts of He-Ne-gas laser.

1. Working Material :- It is a mixture of He-Ne gases in the ratio of 7:1 or 10:1 at a pressure of 1 mm of mercury (Hg) is quartz tube having length 50 cm and diameter of 1.0 cm.

2. Exciting Source :- It is a Radio frequency generator which emit the EM Radiation of frequency $\nu = 4 \times 10^7 \text{ Hz}$ to $4.7 \times 10^{14} \text{ Hz}$ and is used to create a electric discharge in the gas.

Principle and working of He-Ne Laser



Electric discharge :- This is the spark in gases. Source for excitation of gases molecules. by electric discharge in gases occurs when electric current flows through a gases medium due to ionization of the gases.

- When high p.d. is applied in the tube through the Metal bonds.
- He atom get excited to the metastable by means of the population of collision with e^- in the tube.
- Excited the atom collide with Ne atom and the population of Metastable State of Ne atom.
- In this laser action is achieved by Ne atom, He atom only serves the purpose to enhance the excitation process.
- The energy level of the He and Ne Laser is shown as fig. There E_0, E_1, E_2, E_3 are the energy states He atom E_0, E_1, E_2, E_3 are energy state of Ne atoms,
- The accelerated e^- produced by the electric discharge of the gas. collides with the He and Ne atom and excited them to higher state.
- The lifetime of the state E_1 and E_2 for He atom are large about 10^{-4} Sec $\therefore$ He atom accumulated in the level.
- The energy of the state E_3 and E_1 of Ne atom are similar to the energy state of E_2 and E_1 of the He atom.

- The excited He atoms transfer 0.0510 eV energy ($\lambda = 6328 \text{ \AA}$) to the Ne atoms $\therefore$ this transition of $6328 \text{ \AA}$ ($3s^1 \rightarrow 3p^1$) is laser transition. The photons travel to the axis of laser tube, many back and forth b/w Silvered mirrors.
- As a result the photon gets multiple and powerful coherent laser beam emerges from the partial reflector mirror of the tube.

TOPIC :- CO_2 -Laser

PAGE NO

OBJECTIVES:-

OUTCOME:-

Carbon Dioxide Laser :- It is also gas laser, which consist a mixture of three gases CO_2 , N_2 and He here Laser active active by laser four level pumping which properties Laser beam of wave length $9.6 \mu\text{m}$, $10.6 \mu\text{m}$

In CO_2 molecule two oxygen atoms are attached at the two Sides of a carbon atom.

It can have three vibrational Modes of oscillation and molecule vibrates in a Linear combination of these three fundamental modes.

The three mode is

I. Stretched modes

II. Bending modes

III. asymmetric modes. (Asymmetric Stretching).

and Similar nitrogen molecule also possesses the Similar vibration modes.

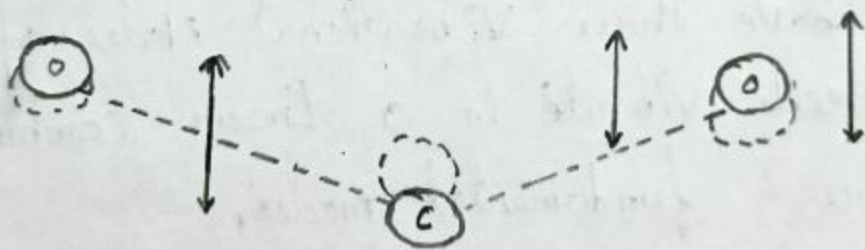
1. Working Substance :- $\text{CO}_2 + \text{Na} + \text{He}$ vapour in ratio 1:2:3 CO_2 is dis is responsible for producing Laser beam area cross-section of discharge tube is 1.5mm^2 and length $\approx 6\text{cm}$..

2. pumping Source :- Electric discharge Method is used for pumping and achieving population inversion.

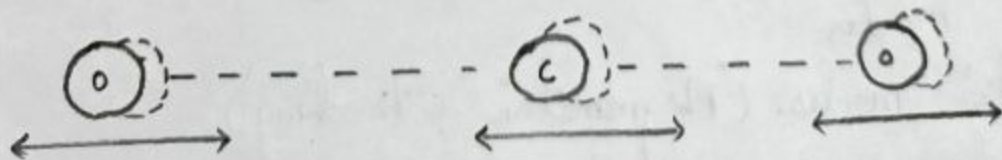
1. Stretched Mode



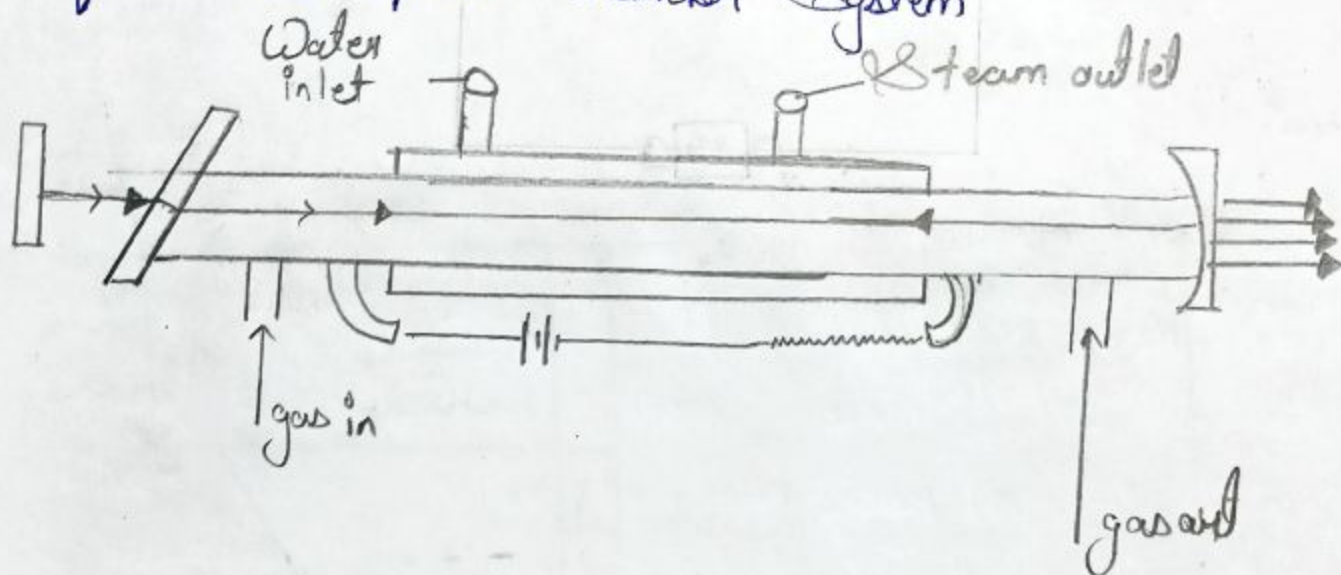
II. Bending Mode.



3. Asymmetric mode.



③. Resonator System :- All the gases mixture is enclosed b/w a set of mirrors which forms the optical resonator system

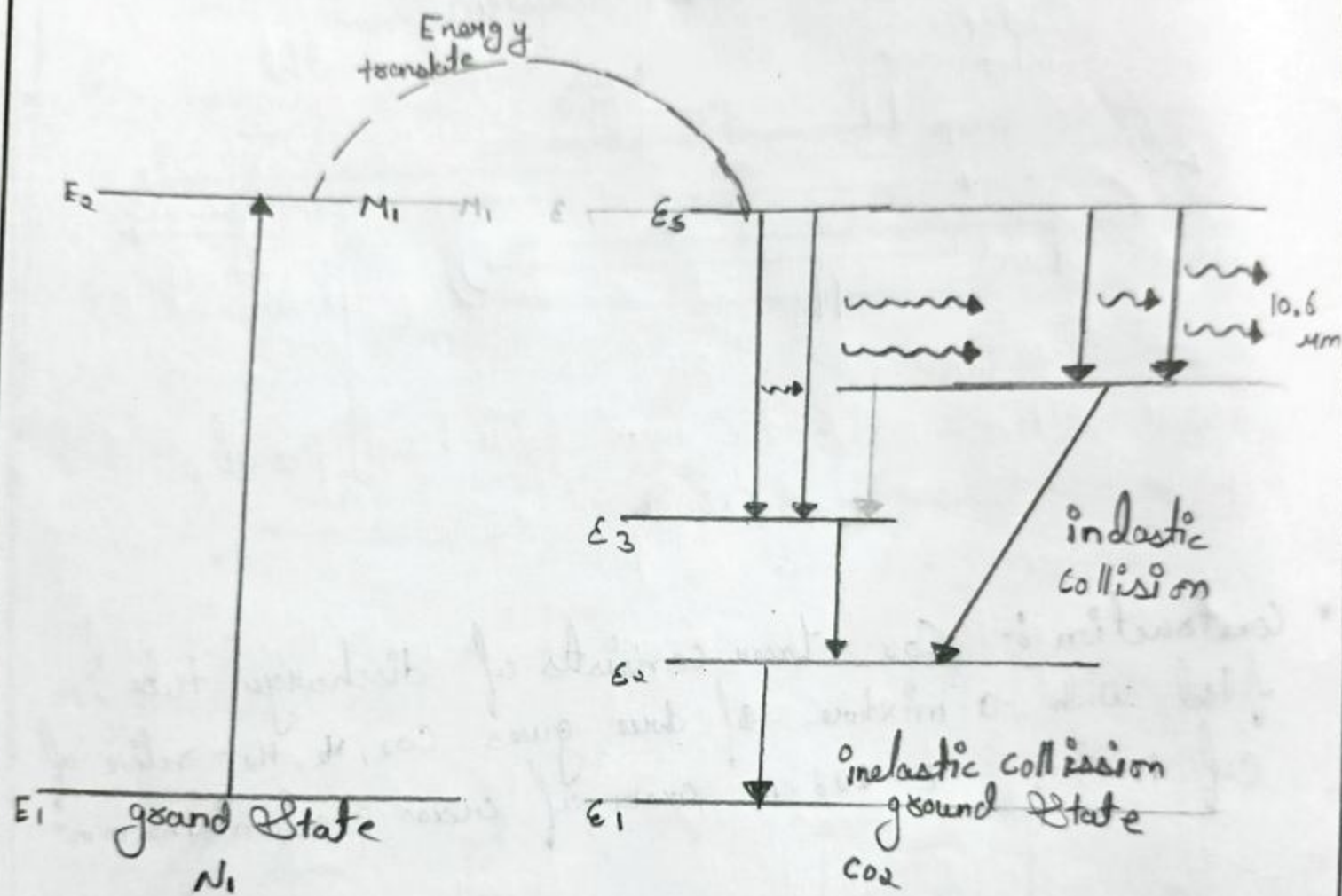


* Construction :- CO_2 Laser consists of discharge tube, filled with a mixture of three gases CO_2 , He , He - ratio of 1:2:3, $L = 826 \text{ cm}$ area of cross section 1.5 mm^2

* Working :- The working of CO_2 laser is same as of He-Ne Laser in which the He is replaced by CO_2 and Ne by Ne .

• When electric discharge is produced in the gas mixture the Ne Molecule get excited to excited state (M)

- In Metastable State molecule stay for quite sometime longer than ordinary excited state because there time about (10^{-3} sec).



REFERENCES:-

SUMMARY:-

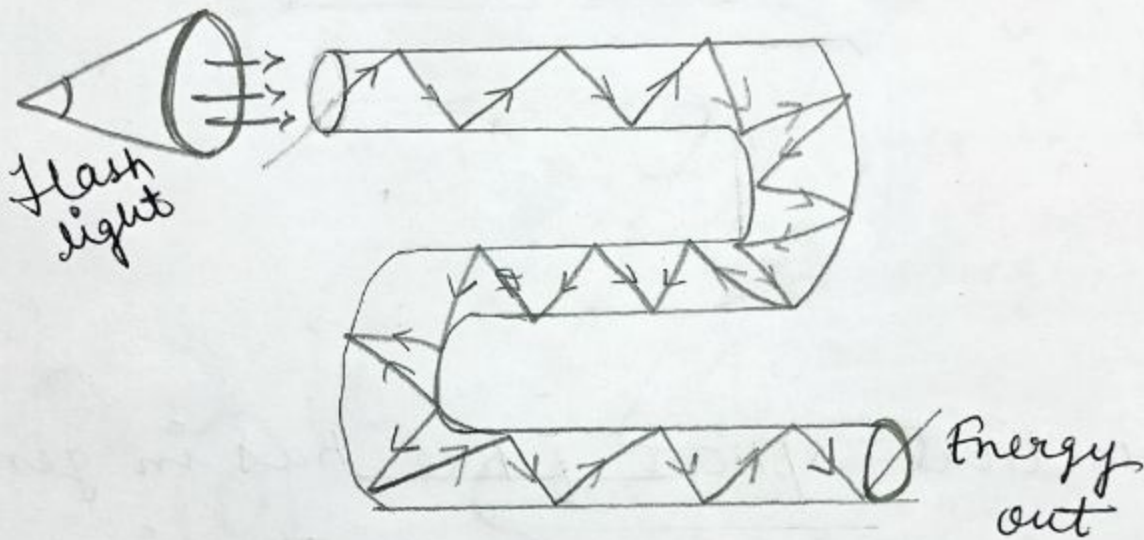
- The no. of Ne molecule in M State gas on $\uparrow$ which is a very favourable condition because the energy of the State equal to the excited State E_5 of Co_2 . E_5 is vibrational level of Co_2 .
- The Ne molecule in excited State (M) transfer its energy to ground State Co_2 molecule through collisions.
- Some Co_2 molecule directly excited to E_5 State through collisions with e^- .
- E_5 is upper lasing level there are two lower lasing level E_4 and E_3 and both transition $E_5 \rightarrow E_4$ and $E_5 \rightarrow E_3$ produces laser light of wavelength 10.6 and 9.6 μ .
- The heating up of the gaseous mixture by electric discharges produce sufficient energy to excite the Co_2 molecules to E_5 level and the mixture is contained by the supply of cold water in the outer jacket.

TOPIC :- Optical fibre and its structure

OBJECTIVES:-

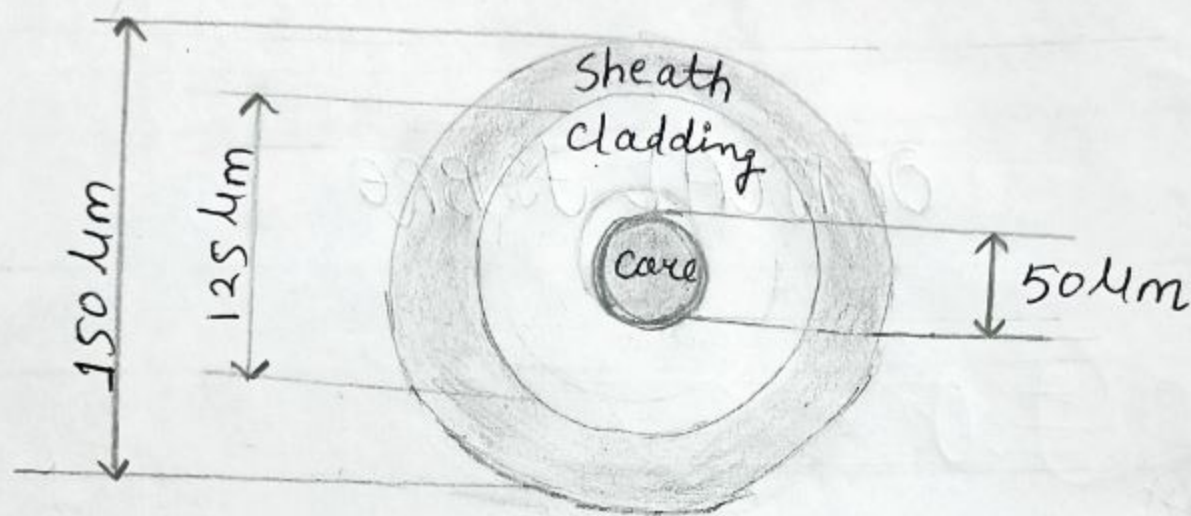
OUTCOME:-

OPTICAL FIBRE



OPTICAL FIBRE :- Optical fibres are glass or plastic conduits as thin as a human hair designed to guide light waves along their length. It is a dielectric waveguide

that operates at optical frequencies.
An optical fibre works on the principle of total internal reflection.

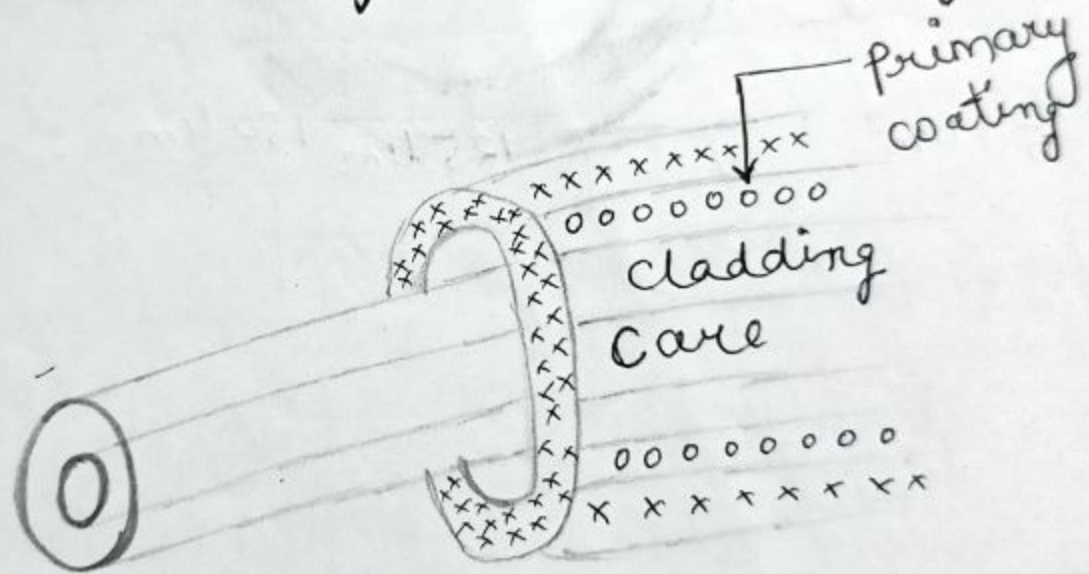


A practical optical fibre has in general three co-axial region. The inner most region is the light guiding region known as the Core. It is surrounded by a co-axial middle region known as the cladding. The outermost region is called a Sheath.

The refractive index of cladding is always lower than that of the core. The purpose of the cladding is to make the light to be confined to the core.

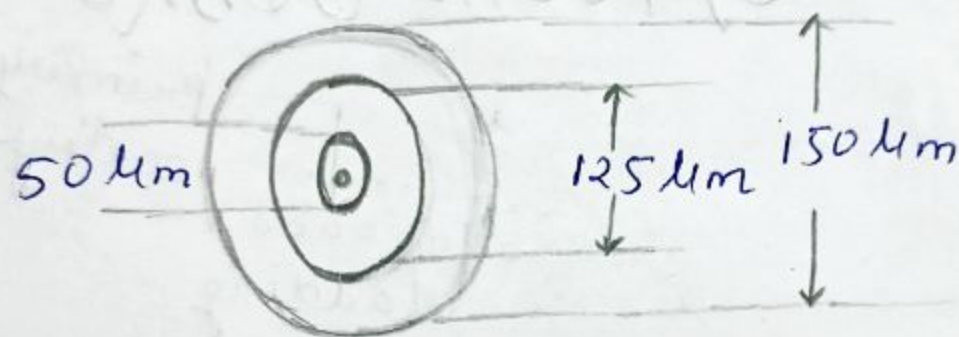
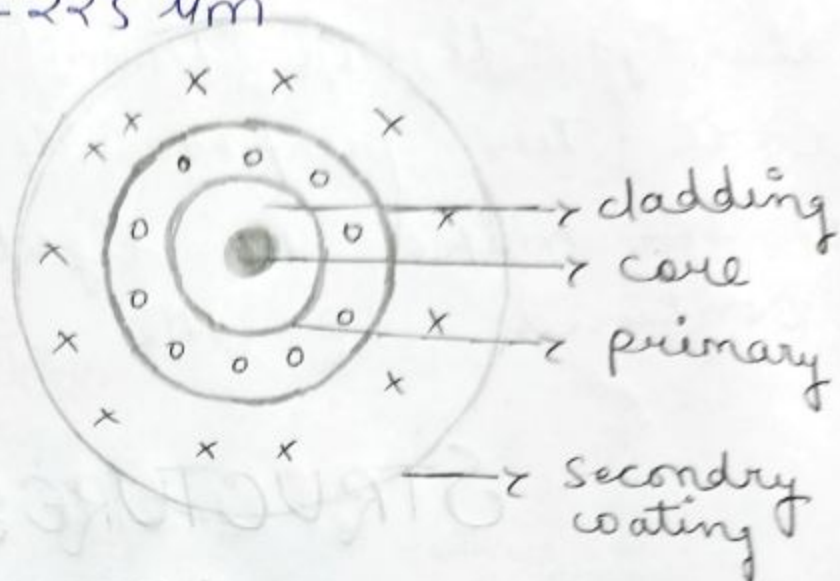
STRUCTURES OF OPTICAL FIBRE

(1)



Diameter of core
 $54\mu\text{m} \approx 100\mu\text{m}$

Sheet 125 μm - 225 μm



REFERENCES:-

SUMMARY:-

Let no. of refractive index of air medium from which light may enter the fibre this end of the fibre is known as launching zone.

Let a ray of light enter the fibre at an angle i to the axis of fibre.

The ray refracts angle r and strike the core cladding interface at an angle θ .

where $\theta > \theta_c$ the light will stay within the fibre.

using snell's Law

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_0}$$

$$n_0 \sin i = n_1 \sin r \quad \dots \dots (1)$$

In triangle BCD

$$r = (90 - \theta)$$

from eqⁿ (1)

$$n_0 \sin i = n_1 \sin (90 - \theta)$$

$$n_0 \sin i = n_1 \cos \theta \quad \dots \dots (A)$$

The Largest value of i is equal to $i_{\max}$
Then,

$$\theta = \theta_c$$

from eqⁿ (A)

$$n_0 \sin i_{\max} = n_1 \cos \theta_c$$

$$\sin i_{\max} = \frac{n_1}{n_0} \cos \theta_c \quad \dots \dots (B)$$

we know that $\sin \theta_c = \frac{n_2}{n_1}$

$$\text{So } \theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

$$\begin{aligned} \cos \theta_c &= \sqrt{1 - \sin^2 \theta_c} \\ &= \sqrt{1 - \left(\frac{n_2}{n_1} \right)^2} \end{aligned}$$

$$\cos \theta_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

from eqⁿ (B)

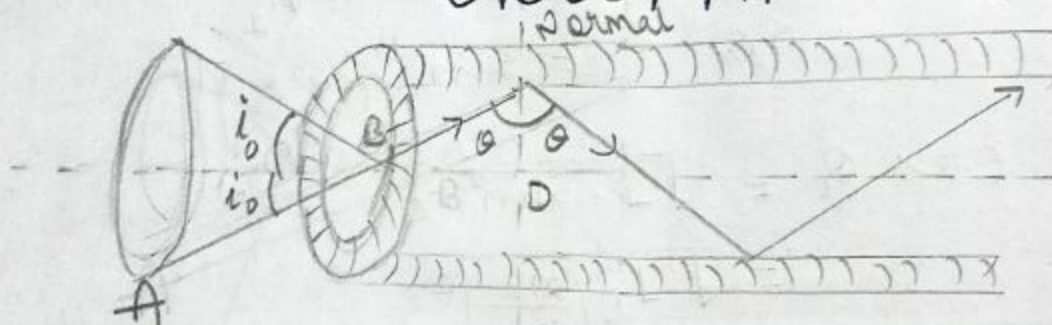
$$\sin i_{\max} = \frac{n_1}{n_0} \cos \theta_c$$

$$= \frac{n_1}{n_0} \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin i_{\max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

Acceptance angle defined as the max^m angle from the fibre axis at which light may enter the fibre so that it propagate in the core by Total Internal Reflection.

ACCEPTANCE CONE



If AB ray is rotated around the fibre axis keeping i_0 it describes a conical surface as shown in figure.

The ray which are funnelled into the fibre having a full angle $2i_0$ will only the only Totally internally reflected and only the ray with in the cone are accepted therefore the cone is called acceptance Cone

REFERENCES:-

SUMMARY:-

TOPIC :- Numerical Aperture and V-Number

OBJECTIVES:-

OUTCOME:-

NUMERICAL APERTURE (NA)

It is determine the light gathering ability of the fibre so it is measure of the amount of light that can be accept by the fibre. This is also defined as sin of acceptance angle.

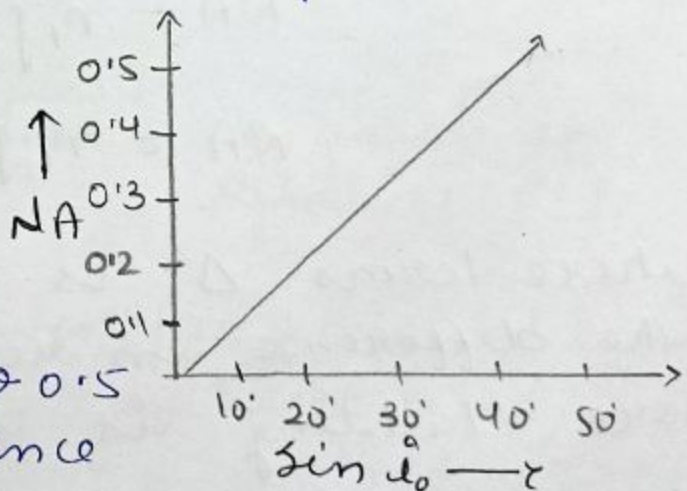
i.e. $NA = \sin \theta_0$

$NA = \sqrt{n_1^2 - n_2^2}$ for air medium

$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$ for any medium except air

In Graphically
The range about
0.1 to 0.3 fibre use
long distance commu-
nication

The range about 0.4 to 0.5
fibre use short distance
communication



Now, NA Evaluating in terms of relative
refractive index

$$\Delta = \frac{n_1 - n_2}{n_1}$$

This is also known as fractional refractive index difference -

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta = 1 - \frac{n_2}{n_1}$$

$$\boxed{\frac{n_2}{n_1} = 1 - \Delta}$$

from eqⁿ $NA = \sqrt{n_1^2 - n_2^2}$

$$NA = n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$NA = n_1 \sqrt{1 - (1 - \Delta)^2}$$

$$NA = n_1 \sqrt{1 - 1 + \Delta^2 + 2\Delta}$$

$$NA = n_1 \sqrt{2\Delta - \Delta^2}$$

$$\boxed{NA = n_1 \sqrt{2\Delta}}$$

where term Δ^2 is neglected because the difference in refractive index of core cladding is very small

V-NUMBER

In case of optical fibre V-Number is determine the operating frequency or normalised which carry out total Internal reflection.

The V-Number is used to calculate the maximum area number of mode (N_m) supported by the optical fibre.

In Mathematically

$$V = \frac{2\pi r}{\lambda} \sin i_0 \quad [\text{in terms of acceptance angle}]$$

$$V = \frac{2\pi r}{\lambda} NA \quad [\text{in terms of numerical aperture}]$$

$$V = \frac{2\pi r}{\lambda} \sqrt{n_1^2 - n_2^2} \quad \text{In terms of R.I core \& cladding}$$

$$V = \frac{2\pi r}{\lambda} n_1 \sqrt{2\Delta} \quad \text{in terms of Relative R.I}$$

where, r = Radius of the fibre

λ = wavelength of light used

n_1, n_2 = R.I of core and cladding

Now the approximate maximum no. of mode in - - -

1) step index fibre

$$N_m = \frac{V^2}{2}$$

2) ^{graded} ~~step~~ index fibre

$$N_m = \frac{V^2}{4}$$

REFERENCES:-

SUMMARY:-

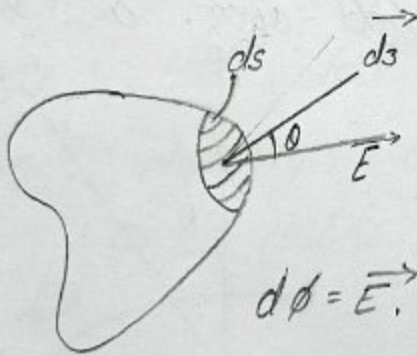
TOPIC :- Electric flux, Coulomb's Law,

OBJECTIVES:-

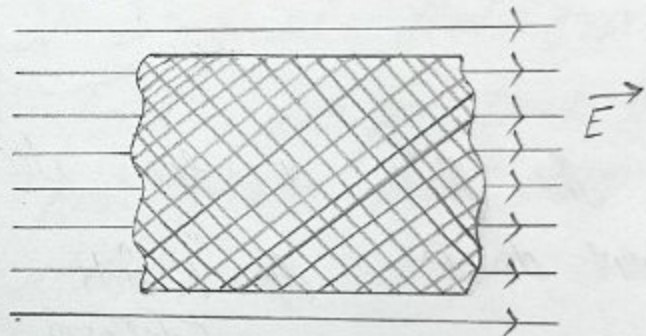
OUTCOME:-

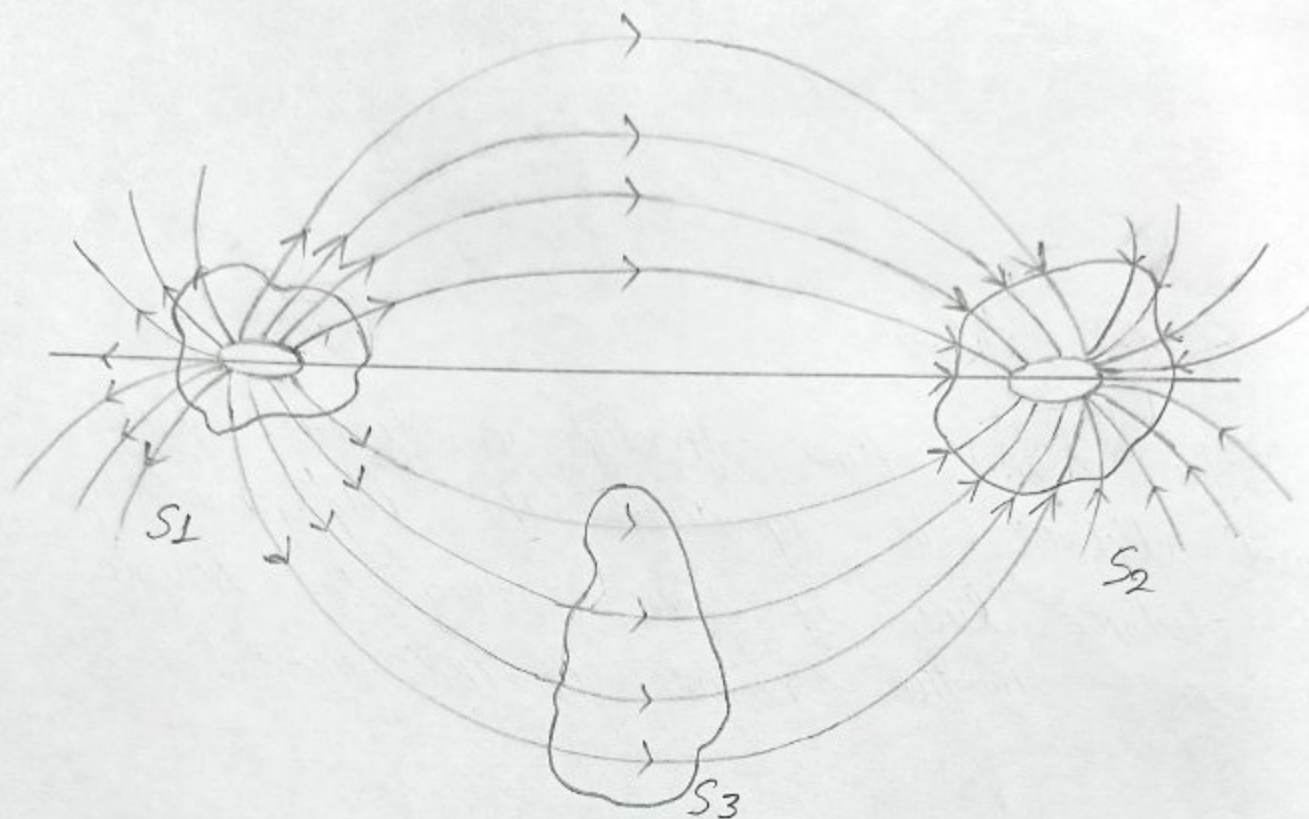
Electric Flux:-

The electric flux through a surface placed inside electric field represented the total number of electric line of force crossing the surface in a direction normal to the surface.



$$d\phi = \vec{E} \cdot \vec{ds}$$





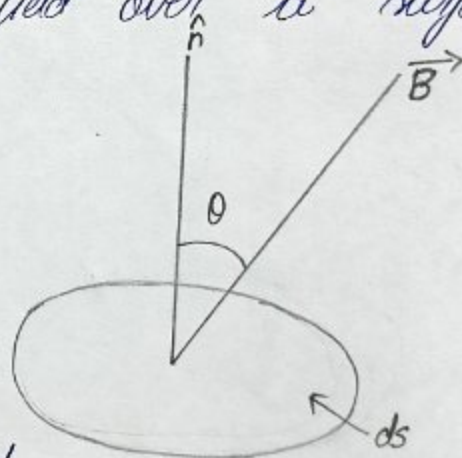
The flux of the electric field over a surface element ds is

$$d\phi = E \cdot \hat{n} ds$$

$$= E ds \cos \theta$$

where

$E \cos \theta$ is the normal component of the field and the normal component represents the number of field lines crossing a unit area.



So, the flux over the number of field lines surface element is actually equal to no. of field lines crossing the area normally.

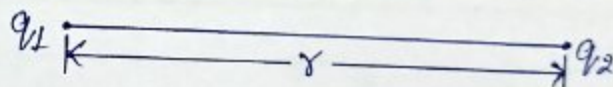
2) And the total flux of the electric field over finite surface

$$\phi = \int_S \vec{E} \cdot \hat{n} ds \quad (\text{vector product}).$$

$$\phi_e = \oint \vec{E} \cdot d\vec{s} \quad (\text{Scalar product}).$$

Coulomb's Law:-

This law gives the definition of the electrostatic force of attraction and repulsion between point charge. It is found that experimentally that b/w two point charges there exist a force that tends to push them pull them together, depending on whether the charges are like or unlike.



Or

"The force b/w two point charges is proportional to the product of charges and is inversely proportional to the square of distance b/w them & acts along the line joining them

$$F \propto \frac{Q_1 Q_2}{R^2} \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{K Q_1 Q_2}{R^2}$$

where K is proportionality constant.

$$\text{where } K = \frac{1}{4\pi\epsilon} \text{ medium}$$

Constant ϵ depends upon the medium or dielectric.

It is called the permittivity of the medium in free space or vacuum $\epsilon = \epsilon_0$

$$\epsilon = \epsilon_0 = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\pi} \text{ (F/m) farad/meter.}$$

$$K = \frac{1}{4\pi\epsilon_0 (\text{vacuum})} = 9 \times 10^9 \text{ m/F}$$

REFERENCES:-

SUMMARY:-

TOPIC :- Gauss theorem

OBJECTIVES:-

OUTCOME:-

Gauss theorem :-

The Electric flux passing through any closed surface is equal to the total charge Enclosed by the surface

$$\phi_e = \frac{1}{\epsilon_0} Q$$

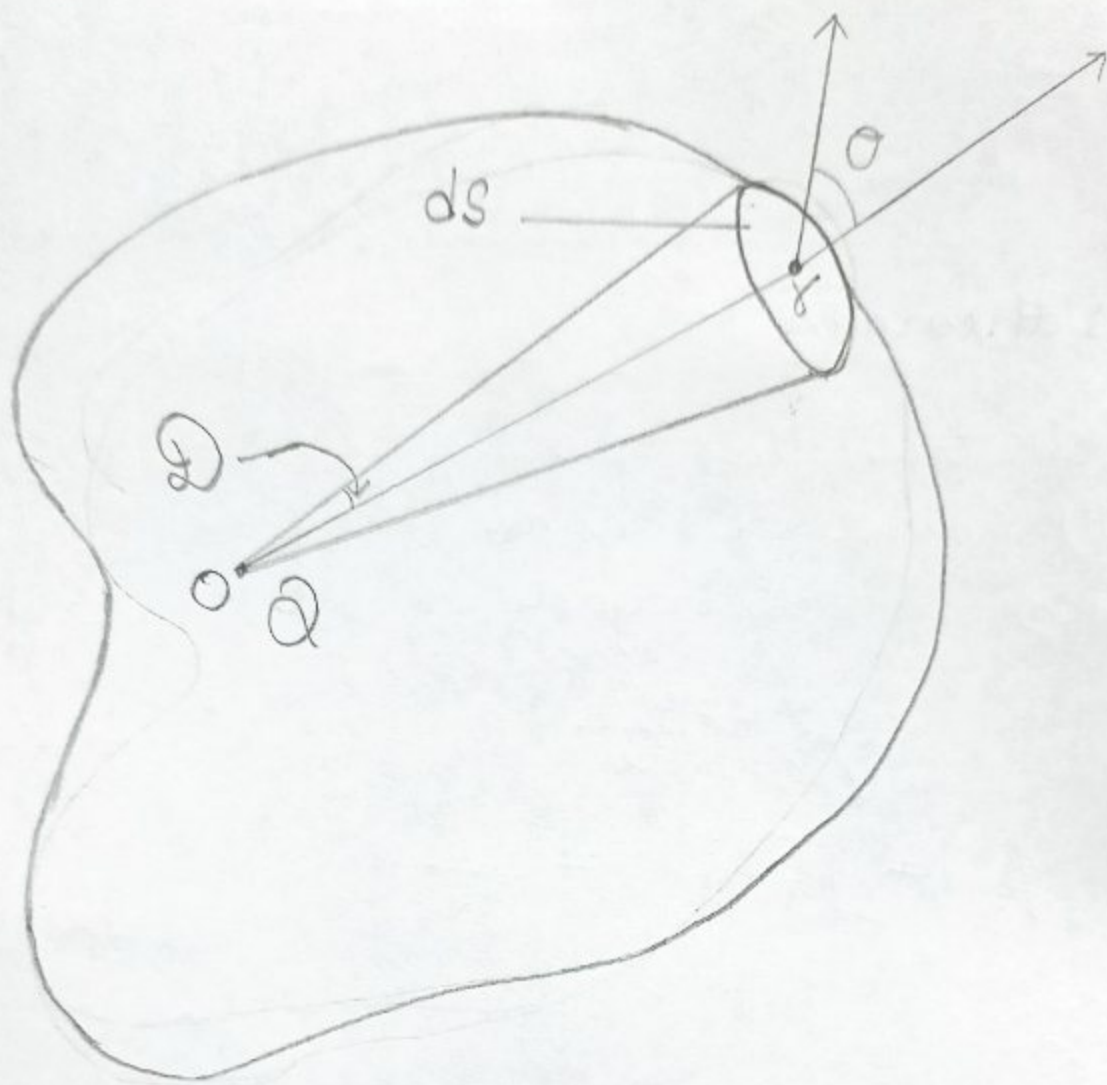
$$\text{or } \phi_e = \oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} Q$$

Proof :-

i) Charge with the fibre

Let a charge take placed at point O within the closed surface a

P on the small surface ds at a distance r from the centre point O



then

$$d\phi_E = \vec{E} \cdot d\vec{S}$$

$$= E ds \cos\theta \quad \text{--- (1)}$$

$$\text{But } \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad \text{--- (2)}$$

$$\Rightarrow d\phi_E = \frac{Q}{4\pi\epsilon_0 r^2} ds \cos\theta$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{ds \cos\theta}{r^2} \right)$$

$$d\phi_e = \frac{Q}{4\pi\epsilon_0} d\omega$$

$$\left[\begin{array}{l} \therefore d\omega = \frac{ds \cos\theta}{r^2} \\ \text{is solid angle} \end{array} \right]$$

then total fluxe over the whole surface taking Integral

$$\int d\phi_e = \frac{Q}{4\pi\epsilon_0} \int d\omega$$

$$\Rightarrow \phi_e = \frac{Q}{4\pi\epsilon_0} 4\pi$$

$$\left[\therefore \int d\omega = 4\pi \right]$$

$$\Rightarrow \phi_e = \frac{1}{\epsilon} Q$$

$$\Rightarrow \boxed{\phi_{\varepsilon} = \frac{1}{\varepsilon} \alpha}$$

REFERENCES:-

SUMMARY:-

TOPIC :- Stokes's theorem

OBJECTIVES:-

OUTCOME:-

Stoke's theorem :-

Statement :-

This theorem states that the line integral of vector field $\vec{A}$ around a closed curve is equal to the surface integral of the curl of $\vec{A}$ taken over the surface S surrounded by the closed curve.

i.e.
$$\oint \vec{A} \cdot d\vec{\ell} = \iint_S \text{curl } \vec{A} \cdot d\vec{S}$$

or
$$\oint \vec{A} \cdot d\vec{\ell} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

Proof :-

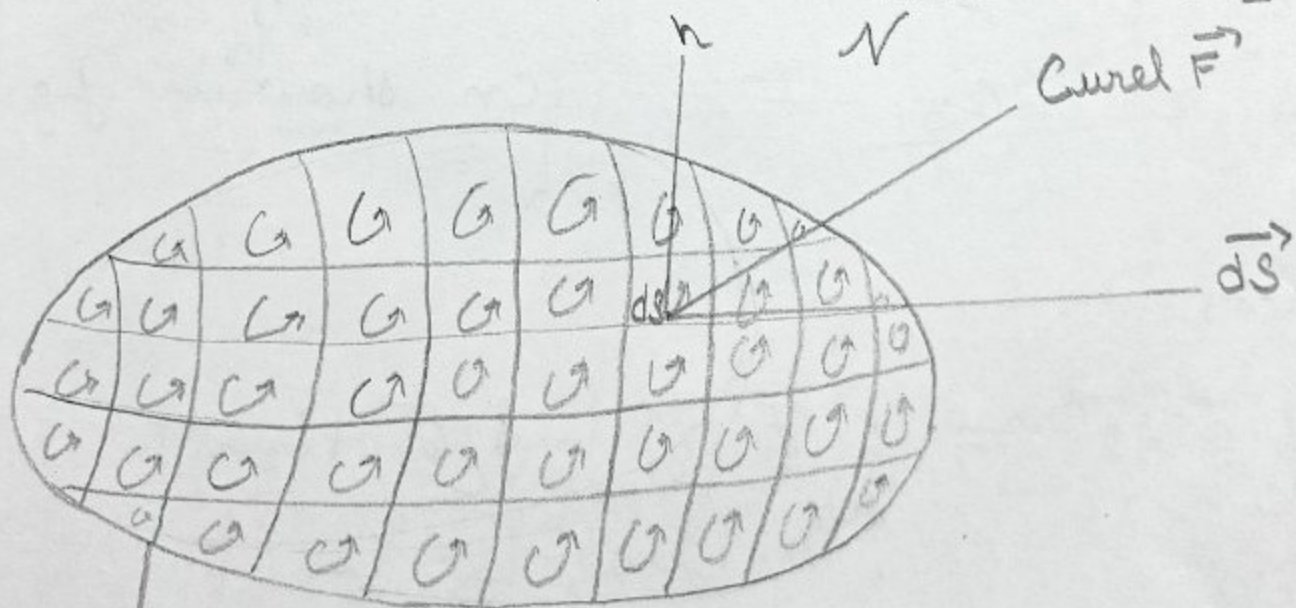
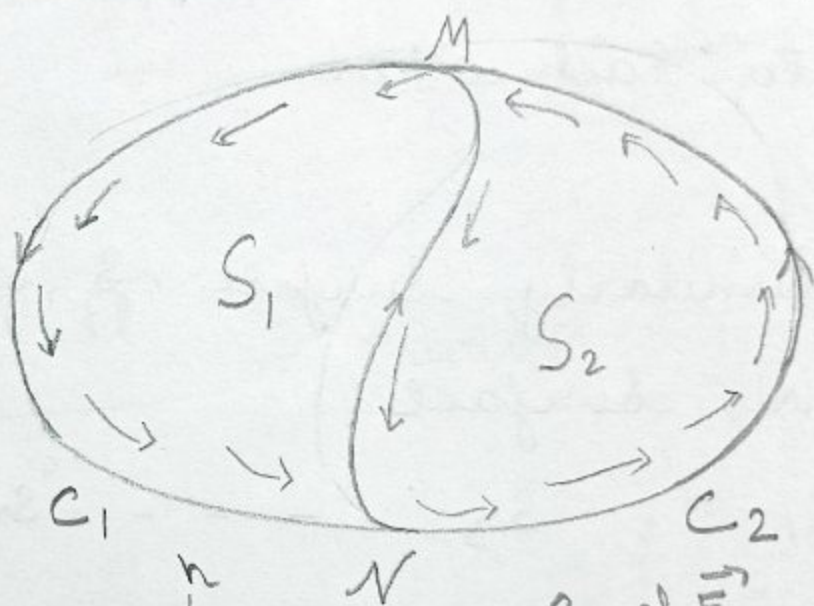
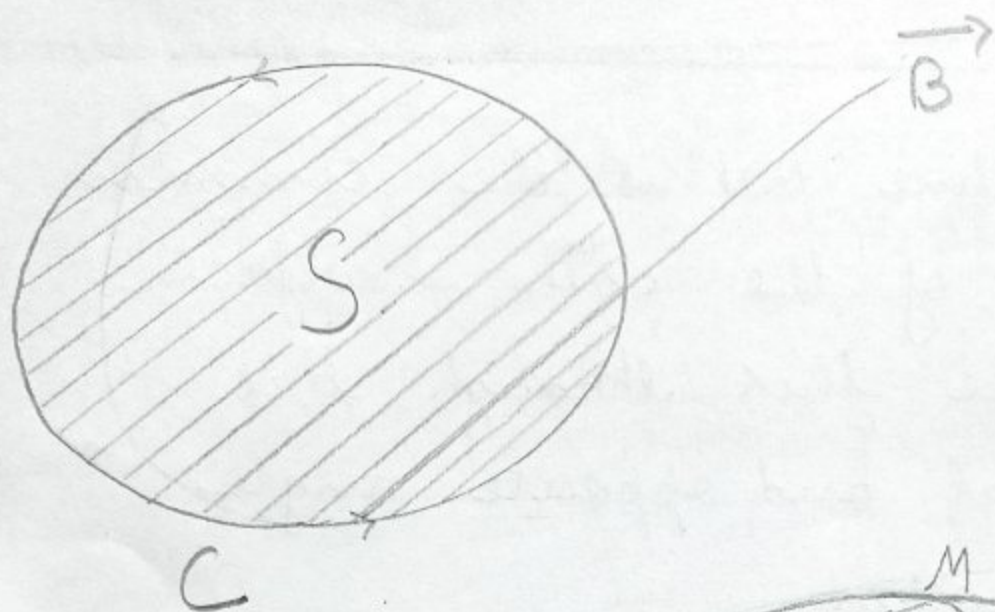
Let us consider a surface S with C its boundary and the line integral of the vector functions around the boundary C of the surface S is

$$\oint_C \vec{F} \cdot d\vec{t}$$

Now surface S divided into two surface S_1 and S_2 which is bounded by C_1 and C_2 then line integral is

$$\oint_C \vec{F} \cdot d\vec{t} = \oint_{C_1} F dl_1 + \oint_{C_2} F dl_2$$

Show in fig



$\downarrow$ CP where $i = 1 \leftarrow N$

In part line MN is the common boundary of the both surface (S_1, S_2) the flux through this line equal and opposite angle to each other.

Similarly surface S divided into N surface

$S_1 \quad S_2 \quad S_3 \quad - - - - S_n$ having boundary

$C_1 \quad C_2 \quad C_3 \quad - - - - C_n$ show in fig

then

$$\oint_C \vec{F} \cdot d\vec{l} = \oint_{C_1} \vec{F} \cdot d\vec{l}_1 + \oint_{C_2} \vec{F} \cdot d\vec{l}_2 + \dots + \oint_{C_n} \vec{F} \cdot d\vec{l}_n$$

$$\Rightarrow \oint \vec{F} \cdot d\vec{\ell} = \sum_{i=1}^N \oint_{C_i} \vec{F} \cdot d\vec{\ell}_i$$

$$\oint \vec{F} \cdot d\vec{\ell} = \sum_{i=1}^N S_i \frac{\oint_{C_i} \vec{F} \cdot d\vec{\ell}_i}{S_i} \quad \text{--- (1)}$$

$$\text{if limit } \frac{\oint_{C_i} \vec{F} \cdot d\vec{\ell}}{S_i} = \text{curl } \vec{F} \quad \text{--- (2)}$$

$$\sum_{i=1}^N S_i = \iint_S d\vec{S} \quad \text{--- (3)}$$

from equation (1) (2) and (3)

$$\oint_C \vec{F} \cdot d\vec{\ell} = \iiint \text{curl } \vec{F} \cdot d\vec{S}$$

This

$$\oint \vec{F} \cdot d\vec{l} = \iint_S \text{curl } \vec{F} \cdot d\vec{S}$$

This is Stoke's theorem

REFERENCES:-

SUMMARY:-

TOPIC :- Gradient

OBJECTIVES:-

OUTCOME:-

Gradient of a scalar field

The gradient of a scalar field ϕ is a vector that gives its magnitude and is directed along the maximum rate of change of ϕ .

Let $\phi(x, y, z)$ scalar function of position of a scalar point of coordinates (x, y, z) . The partial derivatives of ϕ along the three coordinate axes are

$$\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}$$

Then the gradient of scalar function ϕ is

$$\text{grad } \phi = \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z}$$

$$\text{where grad} = \vec{\nabla} = \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$$\Rightarrow \vec{\nabla} \phi = \frac{i d\phi}{dx} + j \frac{d\phi}{dy} + k \frac{d\phi}{dz}$$

where del operator is a vector operator

The gradient is a different operator by means of which are associated a vector field with a scalar field

REFERENCES:-

SUMMARY:-

TOPIC :- Divergence

OBJECTIVES:-

OUTCOME:-

Divergence of a vector field :-

Divergence is a scalar quantities of a vector field

The divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from the point.

The operator ∇ can be involved in the multiplication with a vector.

The scalar or dot product of operator ∇ with a vector $\vec{A}$

i.e $\nabla \cdot \vec{A}$ is called divergence.

then

$$\vec{\nabla} \cdot \vec{A} = \left(\frac{i}{dx} \frac{d}{dx} + y \frac{d}{dy} + k \frac{d}{dz} \right)$$

$$\cdot (iAx + jAy + kAz)$$

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A}$$

$$= \frac{dAx}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz}$$

This is the divergence in Cartesian coordinates.

REFERENCES:-

SUMMARY:-

TOPIC :-

Curl

PAGE NO

OBJECTIVES:-

OUTCOME:-

Curl of a vector field :-

The curl of a vector at any point in a vector field is a vector whose magnitude is equal to the line integral per unit area along the boundary of an infinitesimal area drawn around that point and whose direction is along the normal at this area.

The function $\hat{a}_x \times \frac{d\vec{F}}{dx} + \hat{a}_y \frac{d\vec{F}}{dy} + \hat{a}_z \times \frac{d\vec{F}}{dz}$

is a vector and it is called curl of $\vec{F}$

$$\begin{aligned} \text{i.e. } \text{Curl } \vec{F} &= \hat{a}_x \times \frac{d\vec{F}}{dx} + \hat{a}_y \times \frac{d\vec{F}}{dy} + \hat{a}_z \times \frac{d\vec{F}}{dz} \\ &= \left(\hat{a}_x \frac{d}{dx} + \hat{a}_y \frac{d}{dy} + \hat{a}_z \frac{d}{dz} \right) \times \vec{F} \\ &= \vec{\nabla} \times \vec{F} \end{aligned}$$

i.e. curl of a vector point function $\vec{F}$ is the vector product of the del operation with $\vec{F}$.

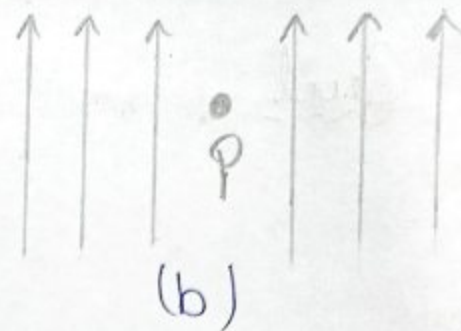
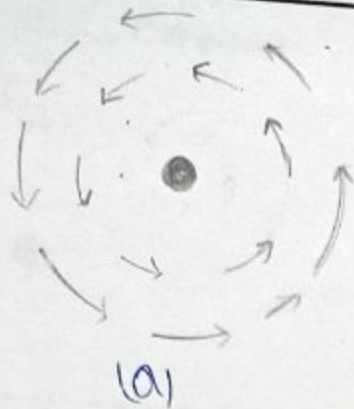


fig (a) shows the curl of a vector field around P which is directed out of the page.

fig (b) shows a vector field with zero curl (is irrotational)

$$\vec{F} = \hat{a}_x f_x + \hat{a}_y f_y + \hat{a}_z f_z$$

$$\text{curl } \vec{F} = \left(\hat{a}_x \frac{d}{dx} + \hat{a}_y \frac{d}{dy} + \hat{a}_z \frac{d}{dz} \right) (\hat{a}_x f_x + \hat{a}_y f_y + \hat{a}_z f_z)$$

REFERENCES:-

SUMMARY:-

TOPIC :- Numerical

PAGE NO

OBJECTIVES:-

OUTCOME:-

#find the value of constant c for which the vector

$$A = (x + 3y)\hat{a}_x + (y + 2z)\hat{a}_y + (x + cz)\hat{a}_z \text{ is solenoidal}$$

$$\text{div } A = 0$$

$$\vec{\nabla} \cdot A = 0$$

$$= \left[\hat{a}_x \frac{d}{dx} + \hat{a}_y \frac{d}{dy} + \hat{a}_z \frac{d}{dz} \right] \cdot$$

$$\left[\begin{array}{c} (x + 3y)\hat{a}_x + (y + 2z)\hat{a}_y + (x + cz)\hat{a}_z \end{array} \right]$$

$$\Rightarrow \frac{d}{dx}(x+3y) + \frac{d}{dy}(y+2z) + \frac{d}{dz}(xy+z)$$

$$\Rightarrow 1+1-c = 0$$

$$\Rightarrow c = -2$$

REFERENCES:-

SUMMARY:-

TOPIC :- Maxwell eqn in differential form.

OBJECTIVES:-

OUTCOME:-

Explanation :-

The electromagnetic equations are known as Maxwell's equations. These equations are basic laws of electricity or magnetism.

In mathematically differential form of Maxwell's equation

$$(1) \quad \text{div } \vec{E} = \rho / \epsilon_0$$

$$\text{or } \vec{\nabla} \cdot \vec{D} = \rho$$

$$(2) \quad \text{div } \vec{B} = 0$$

$$\text{or } \vec{\nabla} \cdot \vec{B} = 0$$

$$(3) \quad \text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\text{or } \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$(4) \quad \vec{\nabla} \times \vec{E} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{or } \text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

(1) Maxwell's first equation

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\text{div } \vec{D} = \rho \quad \text{where}$$

$$D = \epsilon \vec{E}$$

Proof :-

according to Gauss's law in vacuum

$$\phi_E = \oint_S \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

in medium

$$Q_E = \oint_S \vec{E} \cdot d\vec{s} = q/\epsilon \quad \text{--- (1)}$$

where ϵ is permittivity of medium

if charge q inside the close surface is continuously distributed and ρ is the volume charge density

$$\text{then } q = \int_V \rho \, dv \quad \text{--- (2)}$$

from equation (1) and (2)

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon} \int_V \rho \, dv \quad \text{--- (3)}$$

But according to Gauss's law divergence theorem

$$\oint_S \vec{E} \cdot d\vec{s} = \int_V (\vec{\nabla} \cdot \vec{E}) \, dv \quad \text{--- (4)}$$

$$\oint_V (\vec{\nabla} \cdot \vec{E}) dv = \frac{1}{\epsilon} \int_V \rho dv$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon$$

$$\boxed{\text{div } \vec{E} = \rho / \epsilon}$$

in a medium (homogeneous and isotropic)

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon$$

$$\vec{\nabla} \cdot \epsilon \vec{E} = \rho$$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$$

$$\boxed{\vec{D} = \epsilon \vec{E}}$$

$\vec{D}$ is displacement vector

2 Maxwells Second equation

$$\oint_B = 0$$

$$\oint_C \vec{B} \cdot d\vec{s} = 0$$

$$\text{i.e. } \text{div } \vec{B} = 0$$

Proof: according to Gauss's law of magnetism

$$\oint \vec{B} \cdot d\vec{s} = 0 \quad \text{--- ①}$$

the magnetic line force are in the form of closed curve the no incoming and outgoing line of force are same. then net outgoing flux $\oint \vec{B} = 0$

But Gauss's divergence theorem

$$\oint_S \vec{B} \cdot d\vec{s} = \oint_V (\vec{\nabla} \cdot \vec{B}) dv \quad \text{--- (2)}$$

from equation (1) and (2)

$$\oint_V \vec{\nabla} \cdot \vec{B} dv = 0$$

$$\Rightarrow \text{div } \vec{B} = 0$$

This is the different

③ Maxwell Third equation.

According to Faraday's law of electromagnetic law.

when magnetic flux linked with a circuit changes e.m.f is induced in the circuit whose direction is always opposite to the change responsible for its production.

Suppos e.m.f (e) is induced in a closed circuit enclosing a surface S due to changing magnetic flux ϕ_B . [This e.m.f is directly proportional to the rate of change of magnetic flux linked with the circuit]

$$e.m.f = e = - \frac{\partial \phi}{\partial t} \quad \text{--- (1)}$$

where $e = e.m.f$ (electromotive force) ^{negative}

ϕ_B is magnetic flux.

The work done in moving a positive charge around a closed circuit is equal to e.m.f (electromotive force)

[i.e. line integral of electric field $\vec{E}$ is equal to e.m.f]

$$e = \oint_C \vec{E} \cdot d\vec{l} \quad \text{--- (2)}$$

But magnetic flux

$$\phi_B = \oint_S \vec{B} \cdot d\vec{s} \quad \text{--- (3)}$$

from eqⁿ (1) and (3)

$$e = - \frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

$$e = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- (4)}$$

from eq (2) and (4)

$$\oint_C \vec{E} \cdot d\vec{l} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- (5)}$$

Using Stokes's theorem

$$\oint_C \vec{E} \cdot d\vec{l} = \oint_S \text{curl } \vec{E} \cdot d\vec{s} \quad \text{--- (6)}$$

from eq (5) and (6)

$$\oint_S \text{curl } \vec{E} \cdot d\vec{s} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\Rightarrow \oint_S (\text{curl } \vec{E} \cdot d\vec{s} + \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}) = 0$$

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

4 Maxwell fourth equation

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

or

$$\text{curl } \vec{B} = \mu \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right]$$

$$\vec{\nabla} \times \vec{B} = \mu \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right]$$

Proof :- according to Ampere's circular law current is equal to

$$\oint \vec{H} \cdot d\vec{l} = I \quad \text{--- (1)}$$

But according to the definition of current density

$$I = \int_S \vec{J} \cdot d\vec{a} \quad \text{--- (2)}$$

from eqn (1) and (2)

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{a} \quad \text{--- (3)}$$

But According to Stokes's Theorem

$$\oint \vec{H} \cdot d\vec{l} = \int_S \text{Curl } \vec{H} \cdot d\vec{a}$$

from eq (3) and (4)

$$\int_S \text{Curl } \vec{H} \cdot d\vec{a} = \int_S \vec{J} \cdot d\vec{a}$$

$$\int_S [\text{Curl } \vec{H} \cdot \vec{J}] da = 0$$

$$\text{i.e. } \text{Curl } \vec{H} = \vec{J} \quad \text{--- (5)}$$

taking div -

$$\text{div}(\text{Curl } \vec{H}) = \text{div } \vec{J}$$

$$\Rightarrow \text{div } \vec{J} = 0 \quad \text{(6)}$$

This means that the div of current density = zero
This is not for time varying fields.

According to continuity equation.

$$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{--- (7)}$$

from equation (6) and (7) we conclude
that Ampere's eq. (1) is valid only
for steady state condition and is insufficient

for the cases of time varying fields. Hence to include time varying field ampere's Law must be modified.

Maxwell given postulated equation (5) becomes

$$\text{Curl } H = J + J_D \quad \text{--- (8)}$$

taking divergence on both side $\leftarrow$

$$\text{div}(\text{Curl } H) = \text{div}(J + J_D)$$

$$\Rightarrow 0 = \text{div } J + \text{div } J_D$$

from equation (7)

$$\Rightarrow \text{div } J_D - \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \text{div } J_D = \frac{\partial \rho}{\partial t} \quad \text{--- (9)}$$

According to Maxwell first equation

$$\text{div } D = \rho$$

$$\Rightarrow \text{div } J_D = \frac{\partial}{\partial t} (\text{div } D) = \text{div} \left(\frac{\partial D}{\partial t} \right)$$

$$\Rightarrow \text{div } J_D - \text{div} \left(\frac{\partial D}{\partial t} \right) = 0$$

$$J_D = \frac{\partial D}{\partial t} \quad - (10)$$

from equation (9) and (10)

$$\boxed{\text{Curl } H = J + \frac{\partial D}{\partial t}}$$

Proved

TOPIC :- Maxwell's eqⁿ in differential form,

OBJECTIVES:-

OUTCOME:-

Maxwell's Equation :- Maxwell's Equations are a set of 4 complicated equations that describe the world of electromagnetics.

These equations describe how electric and magnetic fields propagate, interact, and how they are influenced by object.

James Clerk Maxwell [1831-1879] was one of the first Scientist to determine the speed of propagation of electromagnetic (EM) waves was the same as the speed of light and hence to conclude the EM waves and visible light were really the same thing.

Few Symbols in the Maxwell's Equation

$$\begin{aligned}\vec{D} &= \epsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \\ \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}\end{aligned}$$

D = Electric flux density

$\vec{E}$ = Electric field intensity

$\vec{H}$ = Magnetic field intensity

$\vec{B}$ = Magnetic flux intensity

μ = permeability

ϵ = permittivity of the medium

σ = conductivity of the medium

ρ = Volume charge density

$\vec{J}_c$ = Conduction current density

$\vec{J}$ = Conventional current density

V = Volume.

• Maxwell's first Equation in Differential form:-
 $\vec{\nabla} \cdot \vec{D} = \rho$ or $\text{div } \vec{D} = \rho$

Proof:- According to Gauss's Law in Vacuum -

$$\phi_E = \frac{q}{\epsilon_0} \quad \text{--- (i)}$$

$$\phi_E = \oint_S \vec{E} \cdot d\vec{S} \quad \text{--- (ii)}$$

from eq. (i) & (ii) -

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad \text{--- (iii)}$$

Where ϵ is Permittivity.

q is the total charge which are distributed in the surface vacuum -

then, $q = \oint_V \rho dv \quad \text{--- (iv)}$

from eq. (iii) & (iv)

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \oint_V \rho dv \quad \text{--- (v)}$$

But according to Gauss's Divergence theorem:-

$$\oint_S \vec{E} \cdot d\vec{s} = \oint_V (\vec{\nabla} \cdot \vec{E}) dv$$

$$\oint_S \vec{E} \cdot d\vec{s} = \oint_V \text{div} \cdot \vec{E} dv \quad \text{--- (vi)}$$

from eqⁿ (vi) & (v) :-

$$\oint_V \text{div} \vec{E} \cdot dv = \frac{1}{\epsilon} \oint_V f dv \quad \text{In Integral form}$$

$$\oint_V \text{div} \vec{E} \cdot dv - \frac{1}{\epsilon} \oint_V f dv = 0$$

$$\oint_V \left[\text{div} \cdot \vec{E} - \frac{f}{\epsilon} \right] dv = 0$$

$$\text{div} \cdot \vec{E} - \frac{f}{\epsilon} = 0$$

$$\text{div} \vec{E} = \frac{f}{\epsilon}$$

or

$$\vec{\nabla} \cdot \vec{E} = \frac{f}{\epsilon}$$

$$\vec{\nabla} \cdot (\epsilon \cdot \vec{E}) = f$$

$$\vec{\nabla} \cdot \vec{D} = f$$

$$\text{div} \cdot \vec{D} = f$$

$\because \vec{D} = \epsilon \cdot \vec{E}$
[Dis displacement factor]

Hence Proved first eqⁿ.

Maxwell's Second Equation in Differential form

$$\Phi_B = 0 \quad \text{or} \quad \oint_S \vec{B} \cdot d\vec{s} = 0$$

Proof :- The magnetic line of force are in the form of closed curve the number of incoming and outgoing line of force are same. then net outgoing flux $\Phi_B = 0$

We know that By Magnetic flux -

$$\Phi_B = \oint_S \vec{B} \cdot d\vec{s} \quad \text{--- (i)}$$

Using Gauss's divergence theorem :-

$$\oint_S \vec{B} \cdot d\vec{s} = \oint_V \text{div } \vec{B} \, dv = 0 \quad \text{--- (ii)}$$

from eq. (i) & (ii) --- $\oint_V \text{div } \vec{B} \, dv = 0$

$$\oint_V \nabla \cdot \vec{B} \, dv = 0$$

$$\boxed{\text{div } \vec{B} = 0}$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

This is the differentiation of second eq.
Hence Proved

REFERENCES:-

SUMMARY:-

TOPIC :- Maxwell's Third Equation

OBJECTIVES:-

OUTCOME:- $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Proof :-

According to Faraday's law of EMF :-
When magnetic flux link with a circuit changes, emf. is induced in the circuit whose direction is always opposite to the change responsible for its production.

Suppose e.m.f. (e) is induced in a closed circuit enclosing a surface S due to change in magnetic flux ϕ_B .

This e.m.f. is directly proportional to the rate of change of magnetic flux linked with the circuit :-

$$e = -\frac{\partial \phi_B}{\partial t} \quad \text{--- (i)} \quad (\phi_B \text{ is magnetic flux})$$

But Magnetic flux :-

$$\phi_B = \oint_S \vec{B} \cdot d\vec{s} \quad \text{--- (ii)}$$

from eqⁿ (i) & (ii) —

$$e = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

$$e = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- (iii)}$$

Now the work done in moving a positive charge around a closed circuit is equal to e.m.f.

[i.e. line integral of electric field $\vec{E}$ is equal to e.m.f.]

$$\mathcal{E} = \oint_L \vec{E} \cdot d\vec{l} \quad \text{--- (iv)}$$

from eq.ⁿ (iii) & (iv) —

$$\oint_L \vec{E} \cdot d\vec{l} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- (v)}$$

Using Stoke's theorem :-

$$\oint_L \vec{E} \cdot d\vec{l} = \oint \text{curl } \vec{E} \cdot d\vec{l} \quad \text{--- (vi)}$$

from eq.ⁿ (v) & (vi) —

$$\oint_S \text{curl } \vec{E} \cdot d\vec{s} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\boxed{\oint_S \text{curl } \vec{E} \cdot d\vec{s} + \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = 0}$$

$$\oint_S \left[\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} \right] \cdot d\vec{s} = 0$$

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\boxed{\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

Hence Proved the third Eq.ⁿ

Maxwell's fourth Equation in Diff. form :-
 $\text{Curl } \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$

Proof :- According to Ampere's circuital law :-

$$I = \oint_L \vec{H} \cdot d\vec{l} \quad \text{--- (i)}$$

But According to the definition of current density-

$$I = \oint_S \vec{J} \cdot d\vec{s} \quad \text{--- (ii)}$$

from eq. (i) & (ii) -

$$\oint_L \vec{H} \cdot d\vec{l} = \oint_S \vec{J} \cdot d\vec{s} \quad \text{--- (iii)}$$

But According to Stoke's theorem :-

$$\oint_L \vec{H} \cdot d\vec{l} = \oint_S \text{Curl } \vec{H} \cdot d\vec{s} \quad \text{--- (iv)}$$

from eq. (iii) & (iv) -

$$\oint_S \text{Curl } \vec{H} \cdot d\vec{s} = \oint_S \vec{J} \cdot d\vec{s}$$

$$\text{Curl } \vec{H} - \vec{J} = 0$$

$$\boxed{\text{Curl } \vec{H} = \vec{J}} \quad \text{--- (v)}$$

taking divergence both side -

$$\text{div}(\text{Curl } \vec{H}) = \text{div } \vec{J}$$

$$\boxed{\text{div } \vec{J} = 0} \quad \text{--- (vi)}$$

that mean's the divergence of current density = zero
 This is not true in time varying fields.

According to Continuity Equation -

$$\boxed{\text{div } \vec{J} = -\frac{\partial \rho}{\partial t}} \quad \text{--- (vii)}$$

From eq. (vi) & (vii) we conclude that ampere's eq. (i) is valid only for steady state conditions and is insufficient for the cases of time varying fields. Hence to include time varying field ampere's law just be modified.

taking div. both side- $\text{Curl } \vec{H} = \vec{J} + \vec{J}_D$ ——— (viii)

$$\text{div}(\text{Curl } \vec{H}) = \text{div}(\vec{J} + \vec{J}_D)$$

$$0 = \text{div}(\vec{J}) + \text{div} \vec{J}_D \quad \left\{ \because \text{div}(\text{curl } \vec{H}) = 0 \right\}$$

$$\therefore \text{div} \vec{J} = -\text{div} \vec{J}_D$$

From eq. (vii) — $+\frac{\partial f}{\partial t} = +\text{div} \vec{J}_D$ ——— (ix)

From Maxwell first eq.ⁿ — $\text{div} \vec{D} = f$

$$\Rightarrow \text{div} \vec{J}_D = \frac{\partial}{\partial t} (\text{div} \vec{D})$$

$$\text{div} \vec{J}_D = \text{div} \left(\frac{\partial \vec{D}}{\partial t} \right)$$

$$\text{div} \vec{J}_D - \text{div} \left(\frac{\partial \vec{D}}{\partial t} \right) = 0, \quad \boxed{\vec{J}_D = \frac{\partial \vec{D}}{\partial t}} \text{ ——— (x)}$$

From eq. (ix) & (x) —

$$\boxed{\text{Curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Hence Proved

REFERENCES:-

SUMMARY:-

TOPIC :- Maxwell's eqⁿ in Integral form

OBJECTIVES:-

OUTCOME:-

The first Maxwell's equation in differential form is

$$\text{div } \vec{D} = \rho = \vec{\nabla} \cdot \vec{D} = \rho$$

Taking volume integral ~~xxx~~ (i).

$$\oint_V \vec{\nabla} \cdot \vec{D} \, dv = \oint_V \rho \, dv \quad \text{--- (ii)}$$

But Gauss's divergence theorem

$$\oint_V \vec{\nabla} \cdot \vec{D} \, dv = \oint_S \vec{D} \cdot d\vec{s} \quad \text{--- (iii)}$$

from eq (ii) & (iii)

$$\oint_S \vec{D} \cdot d\vec{s} = \oint_V \rho \, dv$$

$$\Rightarrow \oint_S \epsilon \vec{E} \cdot d\vec{s} = \oint_V \rho \, dv$$

$$\Rightarrow \boxed{\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon} \oint_V \rho \, dv}$$

$$\text{where } \frac{1}{\epsilon} \oint_V \rho \, dv = q$$

This is the 1st Maxwell Integral eq.

II Maxwell's equation

differential form of m.e.

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{--- ①}$$

taking volume integrating

$$\oint_V \vec{\nabla} \cdot \vec{B} dv = \oint_V 0 dv = 0 \quad \text{--- ②}$$

Gauss's div theorem.

$$\oint_V \vec{\nabla} \cdot \vec{B} dv = \oint_S \vec{B} \cdot d\vec{s} \quad \text{--- ③}$$

of ② & ③

$$\oint \vec{B} \cdot d\vec{s} = 0$$

This is integral form of maxwells second eqn.

Maxwell's Third eqn in Integral form

$$\vec{\nabla} \times \vec{E} = - \partial \vec{B} / \partial t \quad \text{--- ①}$$

taking surface integral

$$\oint_S \vec{\nabla} \times \vec{E} \cdot d\vec{s} = - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- ②}$$

acc to Stokes theorem.

$$\oint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = \oint_C \vec{E} \cdot d\vec{l} \quad \text{--- ③}$$

from eqn ② & ③

$$\oint_C \vec{E} \cdot d\vec{l} = - \oint_S \left(\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \right)$$

This is the integral formula.

Maxwell's fourth equation in integral form

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (1)}$$

taking surface integral

$$\oint_S \vec{\nabla} \times \vec{H} = \oint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) ds \quad \text{--- (2)}$$

Using Stokes theorem —

$$\oint_S \vec{\nabla} \times \vec{H} \cdot d\vec{s} = \oint_C \vec{H} \cdot d\vec{l} \quad \text{--- (3)}$$

From eq (2) & (3)

$$\oint \vec{H} \cdot d\vec{l} = \oint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$$

This is the Maxwell's fourth eq in Integrating form.

This eq signifies that
magnetomotive force (m.m.f.)

$\oint_C \vec{H} \cdot d\vec{l}$ around a closed path is equal to the conduction current plus displacement current through any surface bounded by the path.

$$\vec{B} = \mu \cdot \vec{H}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

TOPIC :- Continuity Equation

OBJECTIVES:-

OUTCOME:-

The continuity equation says that the total current flowing out of some volume must be equal to rate of $\downarrow$ of the charge within that volume.

If charge is neither being created or destroyed we lost since the charge is flowing. Consider that the charge density ρ is function of time

Then the transportation of the charge constitute the current i.e

$$I = \frac{dq}{dt}$$

and

$$q = \int_V \rho \, dv$$

$$\text{So } I = \frac{d}{dt} \int_V \rho \, dv = \int_V \frac{d\rho}{dt} \, dv$$

$$I = \int_V \frac{d\rho}{dt} \, dv \quad \text{--- (1)}$$

Now Current density per unit area normal to the directed surface of volume close by the surface S is

$$I = \oint_S \vec{J} \cdot \vec{ds} \quad - (2)$$

where $\vec{J}$ is current density and $\vec{ds}$ is $\perp$ to S — 1, 2

$$\oint_S \vec{J} \cdot \vec{ds} = - \int_V \frac{d\rho}{dt} dv \quad - (3)$$

(-ve sign given the $\downarrow$ divergence theorem
Charge ρ in volume V when $\uparrow$ the I)

According to Gauss divergence theorem

$$\oint_S \vec{J} \cdot \vec{ds} = \int_V (\text{div } \vec{J}) dv \quad \text{--- (4)}$$

from equation B, 4

$$\int_V \text{div } \vec{J} dv = - \int_V \frac{d\rho}{dt} dv$$

$$\int_V (\text{div } \vec{J} + \frac{d\rho}{dt}) dv = 0$$

$$\Rightarrow \text{div } \vec{J} + \frac{d\rho}{dt} = 0 \quad \text{(A)}$$

This is the continuity equation.

According to Gauss divergence theorem

$$\oint_S \vec{J} \cdot \vec{ds} = \int_V (\text{div } \vec{J}) dv \quad \text{--- (4)}$$

from equation B, or 4

$$\int_V \text{div } \vec{J} dv = - \int_V \frac{d\rho}{dt} dv$$

$$\int_V \left(\text{div } \vec{J} + \frac{d\rho}{dt} \right) dv = 0$$

$$\text{div } \vec{J} + \frac{d\rho}{dt} = 0 \quad \text{--- (A)}$$

This is the continuity equation.

In case of stationary current i.e the Charge density at any point within the region remains constant

$$\text{then } \frac{\partial \rho}{\partial t} = 0$$

from eq (A)

$$\boxed{\text{div } \vec{J} = 0}$$

from above eq. there is no net outward flux of current density $\vec{J}$.

REFERENCES:-

SUMMARY:-

TOPIC :-

Poynting Theorem.

OBJECTIVES:-

OUTCOME:-

Poynting Vector & Poynting Theorem:-

The EM waves carry energy when they propagate and there is an energy density associated with both the electric and magnetic fields.

Poynting Vector :-

The amount of energy flowing through unit area, $\perp$ to the direction of energy propagation per unit time, i.e. the rate of energy transport per unit area, is called the ~~playing~~ Poynting vector.

It is represented by $\vec{S}$

is defined by $\boxed{\vec{S} = \vec{E} \times \vec{H}}$

where $\vec{E}$ is instantaneous value of electric flux.

$\vec{S}$ is, Rate of energy transfer which is $\perp$ to $\vec{E}$ & $\vec{H}$ is in the direction of propagation of the wave.

$\vec{E} \times \vec{H}$ is in the direction of $\vec{k}$ its unit are W/m^2 .

Derivation

Calculate the energy density carried by EM wave by the Maxwell eqn.

$$\text{div } \vec{D} = \rho \quad \text{--- (1)}$$

$$\text{div } \vec{B} = 0 \quad \text{--- (2)}$$

$$\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\text{Curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (4)}$$

Taking (dot) product of eqn (3) & (4) both side with $\vec{H}$ & $\vec{E}$ respectively.

$$\vec{H} \cdot \text{Curl } \vec{E} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \quad \text{--- (5)}$$

$$\vec{E} \cdot \text{Curl } \vec{H} = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \quad \text{--- (6)}$$

Subtract (5)-(6).

$$\vec{H} \cdot \text{Curl } \vec{E} - \vec{E} \cdot \text{Curl } \vec{H} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \vec{J} - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\text{div}(\vec{E} \times \vec{H}) = -\left(\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right) - \vec{E} \cdot \vec{J} \quad \text{--- (7)}$$

$$\because \text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{Curl } \vec{A} - \vec{A} \cdot \text{Curl } \vec{B}$$

$$\text{Now } \vec{B} = \mu \vec{H}, \quad \vec{D} = \epsilon \vec{E}$$

$$\begin{aligned} \text{div}(\vec{E} \times \vec{H}) &= -\left[\vec{H} \cdot \frac{\partial (\mu \vec{H})}{\partial t} + \vec{E} \cdot \frac{\partial (\epsilon \vec{E})}{\partial t}\right] - \vec{E} \cdot \vec{J} \\ &= -\left[\mu \frac{\partial (\vec{H} \cdot \vec{H})}{\partial t} + \epsilon \frac{\partial (\vec{E} \cdot \vec{E})}{\partial t}\right] - \vec{E} \cdot \vec{J} \end{aligned}$$

$$= - \left[\mu \frac{\partial}{\partial t} (H^2) + \epsilon \frac{\partial}{\partial t} (E^2) \right] - \vec{E} \cdot \vec{J}$$

$$\left[= - \left[\frac{\partial}{\partial t} \left\{ \frac{1}{2} (\vec{H} \cdot \vec{H}) \right\} + \frac{\partial}{\partial t} \left\{ \frac{1}{2} (\vec{E} \cdot \vec{E}) \right\} \right] - \vec{E} \cdot \vec{J} \right]$$

$$= - \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \vec{H} \cdot \vec{B} \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{E} \cdot \vec{D} \right) \right] - \vec{E} \cdot \vec{J}$$

$$E^2 = \vec{E} \cdot \vec{E}, \quad H^2 = \vec{H} \cdot \vec{H}$$

$$\text{div}(\vec{E} \times \vec{H}) = - \frac{\partial}{\partial t} \left[\frac{1}{2} (\vec{H} \cdot \vec{B} + \vec{E} \cdot \vec{D}) \right] - \vec{E} \cdot \vec{J}$$

$$\vec{E} \cdot \vec{J} = - \frac{\partial}{\partial t} \left[\frac{1}{2} (\vec{H} \cdot \vec{B} + \vec{B} \cdot \vec{D}) \right] - \text{div}(\vec{E} \times \vec{H})$$

take Volume V of both side.

$$\int_V \vec{E} \cdot \vec{J} \, dv = - \int_V \left[\frac{\partial}{\partial t} \left\{ \frac{1}{2} (\vec{H} \cdot \vec{B} + \vec{E} \cdot \vec{D}) \right\} \right] dv$$

$$- \int_V \text{div}(\vec{E} \times \vec{H}) \, dv$$

$$\Rightarrow \int_V \vec{E} \cdot \vec{J} \, dv = \frac{\partial}{\partial t} \int_V \left[\left(\frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 \right) \right] dv$$

$$- \oint_S (\vec{E} \times \vec{H}) \cdot \vec{ds}$$

where $\vec{E} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$ and

$$\int_V \text{div}(\vec{E} \times \vec{H}) \, dv = \oint_S (\vec{E} \times \vec{H}) \cdot \vec{ds}$$

$$\Rightarrow \int_V \vec{E} \cdot \vec{J} \, dv = - \frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 \right] dv$$

$$- \oint_S (\vec{E} \times \vec{H}) \cdot \vec{ds}$$

This eq is called the Poynting theorem or work energy theorem.

Interpretation

1) $\int_V \vec{E} \cdot \vec{J} \, dv$ is rate of energy transferred into EM field.

2) $\frac{1}{2} \mu H^2$ and $\frac{1}{2} \epsilon E^2$ is energy stored in electric and magnetic field.

and

$\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 \right] dv$ is the rate of $\downarrow$ of energy stored in volume V due to Electric and magnetic field.

3) $\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s}$ is the amount of EM energy crossing the closed surface per second i.e. the rate of flow of outward energy through the surface S enclosing volume V .

and

Vector $(\vec{E} \times \vec{H})$ is known as the Poynting Vector

$\vec{S}$

$$\Rightarrow \vec{S} = (\vec{E} \times \vec{H})$$

Poynting theorem:-

Acc. to which the power transferred into the EM field is equal to the sum of the time rate, of change of EM energy within a certain volume and the time, rate of the energy flowing out through the boundary surface. This is called Poynting theorem and also called the energy conservation law in Electromagnetism.

TOPIC :- Gauss Law in Dielectric

OBJECTIVES:-

OUTCOME:-

Gauss's Law in dielectric

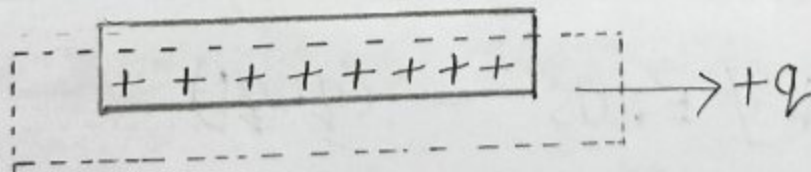
Gauss's law states that "the surface integral of electric field over a closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface i.e.

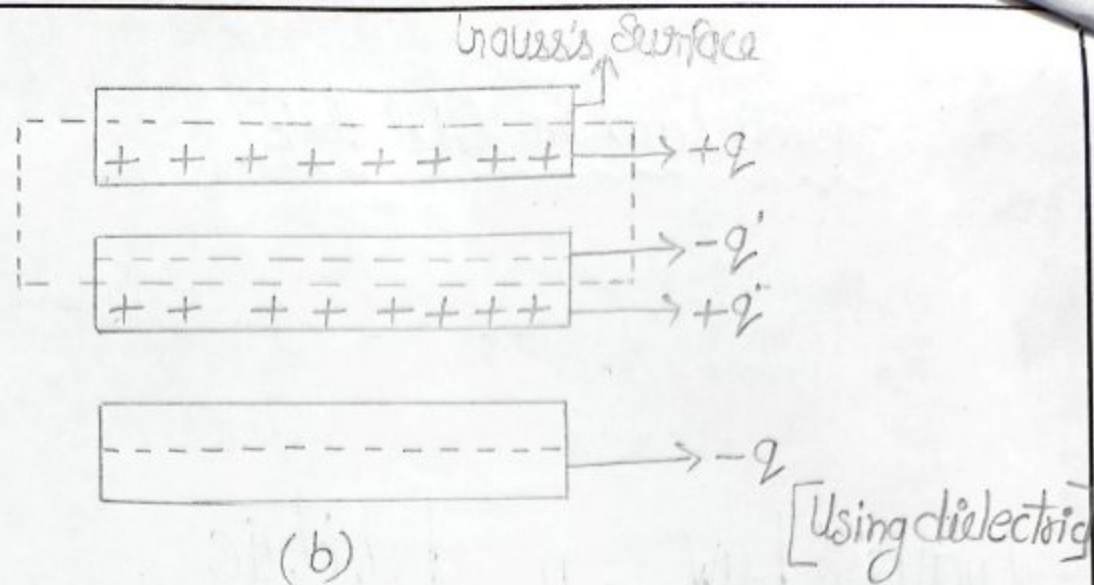
$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} q.$$

$$\Rightarrow \epsilon_0 \oint \vec{E} \cdot d\vec{s} = q$$

$$\Rightarrow \oint \epsilon_0 \vec{E} \cdot d\vec{s} = q$$

Consider a parallel plate capacitor without dielectric as shown in fig.





$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = q$$

$$\Rightarrow \epsilon_0 E_0 S = q$$

$$E_0 = q / \epsilon_0 S \quad \text{--- (1)}$$

Where E_0 Electric flux b/w the plate
 the presence dielectric material b/w capacitor
 plate from fig. (b) total charge enclosed by Gauss's
 surface is $(q - q')$ where q' is induced charge in the dielectric
 due to polarisation then Gauss Law.

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = (q - q') \quad \text{--- (2)}$$

$$\Rightarrow \epsilon_0 E S = q - q'$$

$$E = \frac{q}{\epsilon_0 S} - \frac{q'}{\epsilon_0 S} \quad \text{--- (3)}$$

The relative permittivity is

$$K = \frac{E_0}{E}, \quad E = \frac{E_0}{K} \quad \text{and} \quad \epsilon' = \epsilon_0 K, \quad \epsilon_0 = \epsilon/K$$
$$K = \frac{\epsilon}{\epsilon_0}$$

$$E = \frac{q}{K\epsilon_0 S} \quad - (4)$$

from eqⁿ (3) & (4)

$$\frac{q}{K\epsilon_0 S} = \frac{q}{\epsilon_0 S} - \frac{q'}{\epsilon_0 S}$$

$$\Rightarrow \frac{q}{K} = q - q'$$

The induced charge is

$$\Rightarrow q' = q(1 - 1/K)$$

It is clear that the induced charge q' is less than the free charge q

from eqⁿ (3) & (4)

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = (q - q') = \frac{q}{K}$$
$$= \oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0 K} = \frac{q}{\epsilon}$$

$$\Rightarrow \oint \epsilon \vec{E} \cdot d\vec{S} = q$$

$$\Rightarrow \oint \vec{D} \cdot d\vec{S} = q$$

This is the Gauss's law in the presence of dielectric.

REFERENCES:-

SUMMARY:-

TOPIC :- EM - Maxwell's eqⁿ in free space

OBJECTIVES:-

OUTCOME:-

We know that, Maxwell's equation

$$\vec{\nabla} \cdot \vec{B} = \text{div } \vec{B} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \cdot \vec{B} = \text{div } \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \text{curl } \vec{B} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \text{curl } \vec{H} = \mathbf{J} + \frac{\partial \vec{D}}{\partial t}$$

or,

$$\vec{\nabla} \times \vec{B} = \text{curl } \vec{B} = \mu \left(\mathbf{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

where,

ρ and $\mathbf{J}$ are charge and current density and ϵ and μ are permittivity and permeability of medium.

For free space there is no charge
i.e., $\rho = 0$. so, no current $J = 0$
 $\mu = \mu_0$, $\epsilon = \epsilon_0$.

Therefore maxwell's eqⁿ in free space
reduces to

$$\vec{\nabla} \cdot \vec{E} = 0 \rightarrow \text{eq}^n (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \text{eq}^n (2)$$

$$\vec{\nabla} \times \vec{B} = -\frac{\partial B}{\partial t} \rightarrow \text{eq}^n (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial}{\partial t} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{B} = \epsilon_0 \vec{E}$$

$$\text{or, } \vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow \text{eq}^n (4)$$

(1) WAVE EQⁿ IN TERMS OF $\vec{E}$

From eqⁿ (3)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (1)$$

taking curl of both sides

$$\begin{aligned}\vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} \\ &= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \longrightarrow (2)\end{aligned}$$

$$\begin{aligned}\because \vec{a} \times \vec{b} \times \vec{c} &= b(a \cdot c) - c(a \cdot b) \\ \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \vec{E} (\vec{\nabla} \cdot \vec{\nabla}) \\ &= 0 - \vec{E} \vec{\nabla}^2 \\ &= -\vec{\nabla}^2 \vec{E} \quad \because [\vec{\nabla} \cdot \vec{E} = 0] \\ &\longrightarrow (3)\end{aligned}$$

from eqⁿ (2) and (3)

$$\begin{aligned}-\vec{\nabla}^2 \vec{E} &= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \\ &= -\frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)\end{aligned}$$

$$-\vec{\nabla}^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\vec{\nabla}^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \longrightarrow A.$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

This wave eqⁿ for propagation of electric field $\vec{E}$ in free space

* WAVE EQⁿ IN TERMS OF H

from eqⁿ (4)

$$\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Taking curl

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\Rightarrow -\nabla^2 \vec{H} = \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\Rightarrow -\nabla^2 \vec{H} = \epsilon_0 \frac{\partial}{\partial t} \left(-\mu_0 \frac{\partial \vec{B}}{\partial t} \right)$$

$$\therefore \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \boxed{\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}}$$

—————> B.

This wave eqⁿ for the propagation of $\vec{H}$ in free space.

(3) WAVE EQUATION IN TERMS OF $\vec{B}$

According to Maxwell fourth eqⁿ —

$$\vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Taking curl —

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\Rightarrow -\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}} \longrightarrow (C)$$

(4) WAVE EQUATION IN TERMS OF $\vec{D}$

from eqⁿ (A)

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (1)$$

But,

$$\epsilon_0 \vec{E} = \vec{D}$$

$$\vec{E} = \vec{D} / \epsilon_0$$

$$\nabla^2 (\vec{D} / \epsilon_0) = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} (\vec{D} / \epsilon_0) \quad \text{from eqⁿ (1)}$$

$$\boxed{\nabla^2 \vec{D} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{D}}{\partial t^2}} \longrightarrow (D)$$

Standard form of wave equation :-

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{D} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{D}}{\partial t^2}$$

$$\nabla^2 A = \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2}$$

where, $A = \vec{E} \vec{H} \vec{B} \vec{D}$

v is the velocity of wave eqⁿ

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

μ_0 = permeability of free space = $4\pi \times 10^{-7} \text{ F/m}$

ϵ_0 = permittivity of free space

$$= 8.85 \times 10^{-12}$$

REFERENCES:-

SUMMARY:-